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NONLINEAR DYNAMIC ANALYSIS OF FUNCTIONALLY GRADED SHALLOW SHELLS WITH TIME DEPENDENT PARAMETERS UNDER STATIC LOADING

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This paper deals with research of nonlinear vibration of imperfect shallow shells made of functionally graded materials (FGM) with thickness dependent from time under static and dynamic loadings. The material properties are changing in the thickness direction according to the given power law distribution and the non-linear strain-displacement relationships based on the von Karman theory for moderately large normal deflections. Initial nonlinear system of differential equations transforms to singular ordinary differential equations with variable in time coefficients, which is solved by hybrid perturbation and WKB-Galerkin methods in three steps. Comparison of numerical integration of initial equation and аzproximate analytical solutions are given.

Key words: Asymptotic approach, nonlinear dynamic problem, FGM shallow shells, time dependent parameters.

НЕЛІНІЙНИЙ ДИНАМІЧНИЙ АНАЛІЗ ПОЛОГИХ ОБОЛОНОК ІЗ ФУНКЦІОНАЛЬНО ГРАДІЄНТНИХ МАТЕРІАЛІВ З ПАРАМЕТРАМИ, ЗАЛЕЖНИМИ ВІД ЧАСУ, ПІД ДІЄЮ СТАТИЧНИХ ЗУСИЛЬ

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Стаття присвячена нелінійним коливанням пологих оболонок із функціонально градієнтних матеріалів (ФГМ) під дією статичних зусиль. Властивості матеріалу змінюються в напрямку товщини відповідно до степеневого закону. Нелінійні залежності деформації-переміщення ґрунтуються на теорії Кармана для відносно великих нормальних переміщень. Початкова нелінійна система диференціальних рівнянь трансформується до сингулярного нелінійного диференціального рівняння зі змінними за часом коефіцієнтами, яке вирішується гібридним трикроковим збуренням – ВКБ-Гальоркін методом. Надається порівняння результатів чисельного інтегрування основного рівняння проблеми із запропонованим наближеним аналітичним розв’язком.

Ключові слова: асимптотичний підхід, нелінійна динамічна проблема, ФГМ пологі оболонки, залежні від часу параметри.

НЕЛИНЕЙНЫЙ ДИНАМИЧЕСКИЙ АНАЛИЗ ПОЛОГИХ ОБОЛОЧЕК ИЗ ФУНКЦИОНАЛЬНО ГРАДИЕНТНЫХ МАТЕРИАЛОВ С ПАРАМЕТРАМИ, ЗАВИСЯЩИМИ ОТ ВРЕМЕНИ, ПОД ДЕЙСТВИЕМ СТАТИЧЕСКИХ НАГРУЗОК

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Данная статья посвящена нелинейным колебаниям пологих оболочек из функционально градиентных материалов (ФГМ) под действием статических усилий. Свойства материала изменяются по толщине в соответствии со степеным законом. Нелинейные зависимости деформации-перемещения основаны на теории Кармана для относительно больших перемещений. Исходная нелинейная система дифференциальных уравнений преобразуется к сингулярному нелинейному дифференциальному уравнению с переменными во времени коэффициентами, которое решается гибридным трехшаговым возмущением – ВКБ-Галеркин методом. Дается сравнение результатов численного интегрирования основного уравнения проблемы с предложенным приближенным аналитическим решением.

Ключевые слова: асимптотический поход, нелинейная динамическая проблема, ФГМ пологие оболочки, зависимые от времени параметры.

INTRODUCTION

Thin walled structures made of functionally graded materials (FGM) with metal inner surface and ceramic in outer surface are widely used, for example, in modern air-space systems, shipbuilding, electronics and other fields of science and technology because of their flexibility in design to have desired strength and durability.

FGMs are the heterogeneous composite materials in which the material properties are changing in the thickness direction or discontinuous as a stepwise gradation of the material constituents [1, 2, 4, 11]. Composite shell structures which are used for modern flying apparatus undergo large deflection and/or static and nonlinear dynamic external mechanical loads. That is why it is important to take into account the geometrically nonlinear effects to ensure more accurate structural analysis and design. In recent years, important studies about vibration and stability of FGM plates and shells under static, dynamic loading and in high temperature environment have been carried out, with using mostly numerical approaches [1-6, 17-19] and just few ones deal with analytical-numerical approaches [12-15].

The present paper deals with an approximate analytical solution of nonlinear dynamic problem of FGM imperfect shallow shells based upon the von Karman theory for moderately large normal deflections with time dependent parameters (for example, with thickness depending on time) on the basis of hybrid (P-WKB) asymptotic method, which was successfully applied earlier [7-10, 16, 20, 21].

FORMULATION OF THE PROBLEM. AN APPROXIMATE ANALYTICAL SOLUTION

Suppose the FGM imperfect shallow shell is simply supported at its edges and subjected to a transverse load $q_0(t)$ and compressive edge loads $r_0(t),\ p_0(t)$. With respect to paper [1, 9, 10, 16] it is assumed that modulus of elasticity and the mass density changes in the thickness direction, while the Poisson ratio is assumed to be constant and thickness of shell is function of time. In this analysis, some mechanical properties of shell material are function of time. For this reason, mathematical model became more complicated from the point of view for analytical solution.

In the spirit of [1] consider that initial imperfections in the middle shell surface, a system of nonlinear differential equations for functions of the normal stress and displacement based on the von Karman theory for large deflections are:

$$\begin{gathered}\rho_1\frac{\partial^2 w}{\partial t^2}+\frac{E_1E_3-E_2^2}{E_1\left(1-v^2\right)}\Delta\Delta\left(w-w_0\right)+2\frac{\partial^2\varphi}{\partial x_1\partial x_2}\frac{\partial^2 w}{\partial x_1\partial x_2}-\frac{\partial^2\varphi}{\partial x_2^2}\frac{\partial^2 w}{\partial x_1^2}-\frac{\partial^2\varphi}{\partial x_1^2}\frac{\partial^2 w}{\partial x_2^2}-k_2\frac{\partial^2\varphi}{\partial x_1^2}-k_1\frac{\partial^2\varphi}{\partial x_2^2}=q_0,\\ \frac{1}{E_1}\Delta\Delta\varphi=-k_1\frac{\partial^2\left(w-w_0\right)}{\partial x_2^2}-k_2\frac{\partial^2\left(w-w_0\right)}{\partial x_1^2}+\left[\left(\frac{\partial^2 w}{\partial x_1\partial x_2}\right)^2-\frac{\partial^2 w}{\partial x_1^2}\cdot\frac{\partial^2 w}{\partial x_2^2}\right]-\left[\left(\frac{\partial^2 w_0}{\partial x_1\partial x_2}\right)^2-\frac{\partial^2 w_0}{\partial x_1^2}\cdot\frac{\partial^2 w_0}{\partial x_2^2}\right]=0,\end{gathered}\tag{1}$$

where q_0 is intensity of transverse load, φ is stress function *w* is deflection.

For the given boundary conditions the deflection function $w=\left(x_1,x_2,t\right)$ is chosen here as

$$w\left(x_1,x_2,t\right)=f\left(t\right)\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}.\tag{2}$$

The regular initial imperfection of middle surface of shell are taken in the form

$$w_0\left(x_1,x_2\right)=f_0\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b},\tag{3}$$

where f_0 is the amplitude of initial imperfection.

Then, applying the Bubnov-Galerkin procedure to the Karman equations (1), we obtain the non-linear second order ordinary differential equation with variable in time coefficients for function $f(t)$ in the following form`:

$$\begin{gathered}\varepsilon^2\frac{d^2f}{dt^2}+f\left(1+2f_0\bar{A}_2\left(t\right)-\bar{A}_3\left(t\right)f_0^2-\bar{A}_1\left(t\right)\right)+f^2\left(-3\bar{A}_2\left(t\right)\right)+f^3\bar{A}_3\left(t\right)=\\ =Q_0-\bar{A}_0\left(t\right)+f_0-\bar{A}_2\left(t\right)f_0^2,\end{gathered}\tag{4}$$

where

$$\begin{gathered}\varepsilon^2=\frac{1}{\omega_{mn}^2},\quad A_0\left(t\right)=\frac{16h\left(t\right)}{\pi^2mn}\left(k_1r_0+k_2p_0\right),\quad A_1\left(t\right)=\frac{\pi^2h\left(t\right)}{a^2}\left(m^2r_0+n^2\lambda^2p_0\right),\\ A_2\left(t\right)=\frac{16E_1\left(t\right)mn\lambda^2\left(k_1n^2\lambda^2+k_2m^2\right)}{3a^2\left(m^2+n^2\lambda^2\right)^2},\quad A_3\left(t\right)=\frac{512E_1\left(t\right)m^2n^2\lambda^4}{9a^4\left(m^2+n^2\lambda^2\right)^2},\quad \bar{A}_i=\frac{A_i}{\omega_{mn}^2},\end{gathered}\tag{5}$$

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$$\omega_{mn}^2 = \frac{1}{\rho_1(t)}\left[\frac{\left(E_1E_3 - E_2^2\right)}{E_1\left(1-\nu^2\right)}\cdot\frac{\left(m^2+n^2\lambda^2\right)\pi^2}{a^4} + \frac{E_1\left(k_1n^2\lambda^2 + k_2m^2\right)^2}{\left(m^2+n^2\lambda^2\right)^2}\right],$$

$$E(t) = \left(E_m + \frac{E_c - E_m}{k+1}\right)h(t), \quad \rho = \left(\rho_m + \frac{\rho_c - \rho_m}{k+1}\right)h(t), \quad \nu(z) = const,$$

where k_1, k_2 are curvatures of middle surface shell in x_1 and x_2 directions, $E(t)$ is Young's modulus, ρ is mass density, $\nu(z)$ – Poisson's ratio, $h(t)$ is thickness of construction, which is depends of time. The volume-fractions of the metal and ceramic phases base by power law $V_m + V_c = 1$, (m – belongs to metal, c – to ceramic), V_c can be express as $V_c = \left(\frac{2z + h(t)}{2h(t)}\right)^k$, k is the volume-fraction exponent $(k \geq 0)$.

Basic differential equation (4) is rewritten in the form

$$\varepsilon^2\frac{d^2 f}{dt^2} + B_1(t) f + \mu\left(B_2(t) f^2 + B_3(t) f^3\right) = \bar{Q}_0(t), \tag{6}$$

where

$$B_1(t) = 1 + 2 f_0\bar{A}_2(t) - \bar{A}_3(t) f_0^2 - \bar{A}_1(t), \quad B_2(t) = \frac{-3}{\mu}\bar{A}_2(t), \quad B_3(t) = \frac{1}{\mu}\bar{A}_3(t), \tag{7}$$

$$\bar{Q}_0(t) = Q_0 - \bar{A}_0(t) + f_0 - \bar{A}_2(t) f_0^2,$$

ε, μ are small parameters.

According to the perturbation method with respect to the parameter of nonlinearity μ, a solution of the differential equation (6) is presented in the form of the following two terms approximation [7]:

$$f(t) = \varphi_0(t) + \mu\varphi_1(t). \tag{8}$$

Substituting (7) into equation (5) and acquainted the terms with the same order of the small parameter we obtain the system equations for unknown functions $\varphi_0(t)$ and $\varphi_1(t)$:

$$\mu^0: \quad \varepsilon^2\varphi_0''(t) + B_1(t)\varphi_0 = \bar{Q}_0, \tag{9}$$

$$\mu^1: \quad \varepsilon^2\varphi_1''(t) + B_1(t)\varphi_1 = -B_2(t)\varphi_0^2 - B_3(t)\varphi_0^3. \tag{10}$$

The system of ordinary singular differential equations with variable in time coefficient B_1 is solved by two terms WKB-approximation [5]. Finally, we have obtained the solution of nonlinear problem on the basis of the perturbation two-terms WKB method as

$$\begin{aligned} f(t) &= \varphi_0(t) + \mu\varphi_1(t) = \sin K(t)\left(c_1 + \bar{c}_1(t)\right) + \cos K(t)\left(c_2 + \bar{c}_2(t)\right) + \\ &\quad + \mu\left(\sin K(t)\left(d_1 + \bar{d}_1(t)\right) + \cos K(t)\left(d_2 + \bar{d}_2(t)\right)\right) = \\ &= B_1(t)^{0.25}\left\{sinK(t)\left[c_1 + \bar{c}_1(t) + \mu\bar{d}_1(t)\right] + cosK(t)\left[c_2 + \bar{c}_2(t) + \mu\bar{d}_2(t)\right]\right\}, \end{aligned} \tag{11}$$

where

$$K(t) = \int \varepsilon^{-1} B_1^{\frac{1}{2}}(t)dt, \tag{12}$$

$$\bar{c}_1(t) = \varepsilon\int\frac{\bar{Q}_0(t)\cos K(t)}{B_1^{-0.25}(t)}dt, \quad \bar{c}_2(t) = -\varepsilon\int\frac{\bar{Q}_0(t)\sin K(t)}{B_1^{-0.25}(t)}dt, \tag{13}$$

$$\bar{d}_1(t)=\varepsilon\int\frac{\left(-B_2(t)\varphi_0^2-B_3(t)\varphi_0^3\right)\bar{Q}_0(t)\cos K(t)}{B(t)^{-0.25}}dt\,, \tag{14}$$

$$\bar{d}_2(t)=-\varepsilon\int\frac{\left(-B_2(t)\varphi_0^2-B_3(t)\varphi_0^3\right)\bar{Q}_0(t)\sin K(t)}{B_1(t)^{-0.25}}dt\,. \tag{15}$$

Initial conditions are taken in the following form:

$$\begin{aligned}\varphi(0)&=1,\\ \varphi'(0)&=0.\end{aligned} \tag{16}$$

In the third step on the hybrid (P-WKB-G) asymptotic method we will keep the perturbation functions but replaced the gauge functions by new amplitudes which depend on ε. In the Bubnov-Galerkin orthogonally principle one seeks an approximate solution in the form of a linear combination of specified (known) coordinate functions with unknown amplitude δ_0 which is a function of ε:

$$f_H(t,\varepsilon)=\exp\int\delta_0(\varepsilon)\varphi_0(t)\,dt\,, \tag{17}$$

where

$$\delta_{01,2}=-\frac{\bar{Q}_0(b)-\bar{Q}_0(a)}{\pm4\int_a^b\left(i\bar{Q}_0(t)^{\frac{3}{2}}\right)dx}\pm\sqrt{\left(\frac{\bar{Q}_0(b)-\bar{Q}_0(a)}{4\int_a^b\pm\left(i\bar{Q}_0(t)^{\frac{3}{2}}\right)dx}\right)^2-\frac{1}{\varepsilon^2}}\,. \tag{18}$$

The result of three-step hybrid asymptotic solution of initial non-homogeneous nonlinear differential equation with variable coefficients for function $f^H(t)$ is given in the following form:

$$\begin{aligned}f^H(t)=B_1(t)\Big\{&\sin I^H\left(t,\delta_{0_1}\right)\left[c_1+\bar{c}_1\left(\bar{Q}_0(t),N(f_0)\right)\right]+\\ &+\cos I^H\left(t,\delta_{0_2}\right)\left[c_2+\bar{c}_2\left(Q_0(t),N(f_0)\right)\right]\Big\},\end{aligned} \tag{19}$$

where

$$I^H(t)=\pm\int\delta_{0_{1,2}}iQ_0(t)^{1/2}\,dt\,. \tag{20}$$

INFLUENCE OF STATIC LOADING AND INITIAL IMPERFECTION OF THE MIDDLE SHELL SURFACE

In this section, we presented the influence of static loading and imperfections of the middle shell surface, where thickness given in form (21). On the Figure 1 there is comparison of asymptotic solution with direct numerical calculation of initial equation, where parameter of static loading is 0.5, amplitude of imperfection is 0.1. On the Figure 2 there is a mold of imperfect shell under static loading, where forced vibration function and parameter a are given by relations (22), (23) respectively. Shapes of shell vibrations for different wave numbers are presented on the Figure 3-9.

$$h(t)=h_0(1-\eta t)\,, \tag{21}$$

$$Q_0=Sin\Omega t\,, \tag{22}$$

$$a=k_1p_0+k_2r_0\,. \tag{23}$$

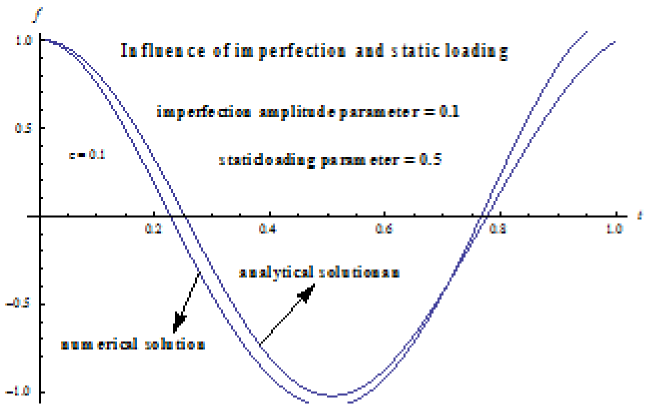

Fig. 1. Comparison of analytical and numerical solutions

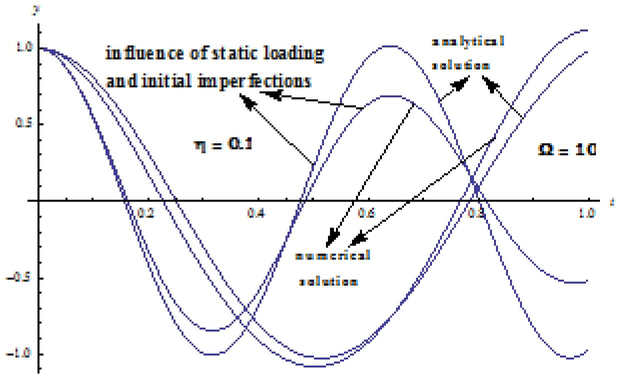

Fig. 2. Influence of static loading and initial imperfection parameters

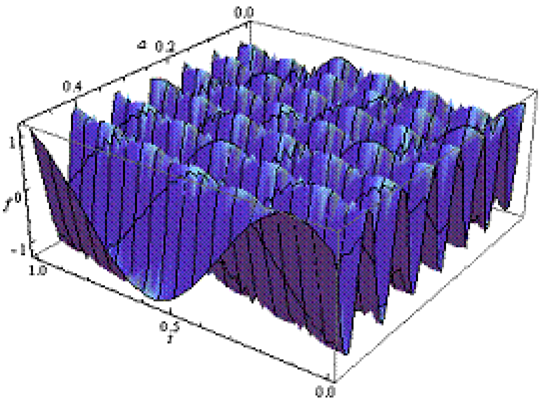

Fig. 3. Vibration mold of imperfect shell under static loading

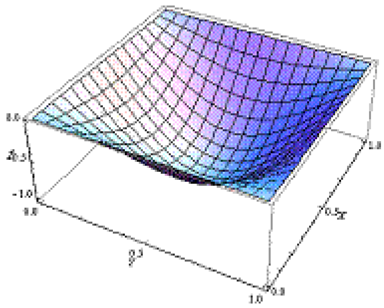

Fig. 4. Shapes of shell vibrations for:

$m=1,\ n=1,\ t=0.5,\ a=0.1$

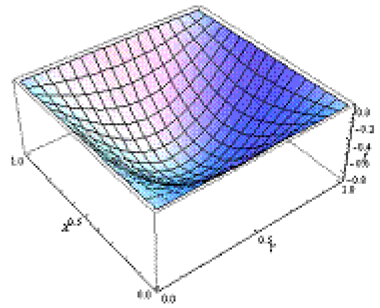

Fig. 5. Shapes of shell vibrations for:

$m=1,\ n=1,\ t=0.5,\ a=0.2$

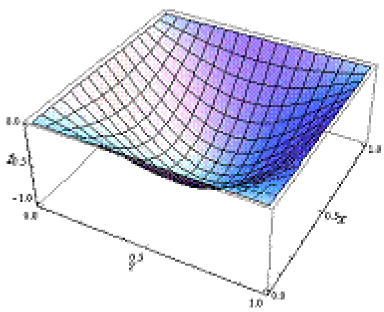

Fig. 6. Shapes of shell vibrations for:

$m=1,\ n=1,\ t=0.5,\ a=0.25$

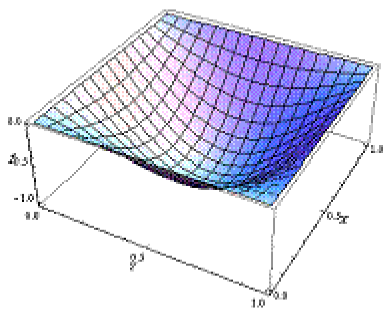

Fig. 7. Shapes of shell vibrations for:

$m=1,\ n=1,\ t=0.5,\ a=0.5$

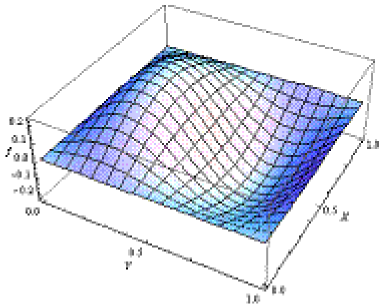

Fig. 8. Shapes of shell vibrations for: $m = 2,\ n = 1,\ t = 0.5,\ a = 0.1$

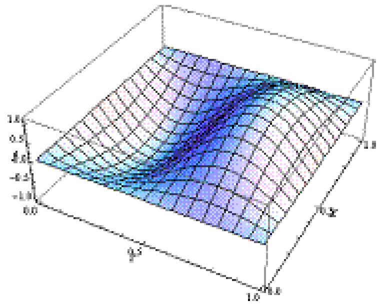

Fig. 9. Shapes of shell vibrations for: $m = 2,\ n = 1,\ t = 0.5,\ a = 0.5$

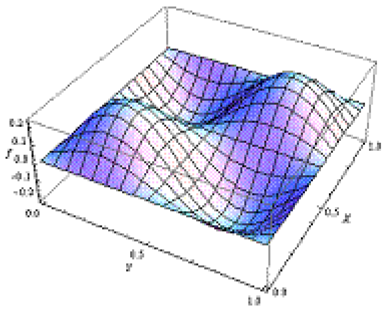

Fig. 10. Shapes of shell vibrations for: $m = 2,\ n = 1,\ t = 0.5,\ a = 0.8$

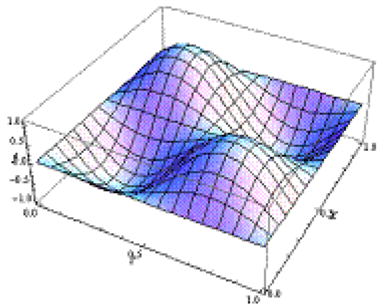

Fig. 11. Shapes of shell vibrations for: $m = 2,\ n = 1,\ t = 0.5,\ a = 0.6$

CONCLUSIONS

An approximate analytical solution for forced vibrations of non-linear FGM shallow cylindrical shells with time dependent thickness on the basis of hybrid perturbation-two-terms WKB approximation method are obtained. In particularly influence of static loading parameter leads to changing of vibration shapes of structure. For some parameters of structure, analytical solutions are in a good enough correlations with direct numerical simulation of initial nonlinear differential equation with variable in time coefficients. Further investigations will be devoted to analyze the nonlinear dynamic behavior of complex shape FGM shell including temperature effects.

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