UDC 539.3
DOI https://doi.org/10.26661/2413-6549-2021-1-07

THE DETERMINATION OF THE ELASTIC CONSTANTS OF THE COMPOSITE MATERIAL WITH SOLID AND HOLLOW EQUIVALENTLY DIRECTED FIBERS

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Key words: *composite material, solid fibers, hollow fibers, effective elastic constants, homogenization, transtropic material.*

The article presents approaches to determining the effective mechanical properties of a composite material reinforced with solid and hollow fibers, using the method of a representative bulk element. Matrix and fiber materials were considered transtropic. The mutual arrangement of solid and hollow fibers is periodic in the general hexagonal scheme of reinforcement. Double homogenization was used to determine the effective elastic characteristics of a composite material containing areas with two types of fibers. The entire volume of the composite is divided into a system of hexagonal areas, of which two types can be distinguished. The first is a solid fiber with its surrounding matrix, the second is a hollow fiber with its surrounding matrix. To pre-homogenize each type of inhomogeneous area, the method of a representative bulk element is used. As a result, there are homogenized areas consisting of hexagonal cells of two types, each of which is transtropic. The isotropy planes for both regions coincide. Taking into account the periodic nature of reinforcement, homogenized regions with solid fibers can be represented as a conditional fiber, and homogenized areas with hollow fibers as a conditional

matrix. We have an inhomogeneous material with a conditional transtropic matrix and fiber. The inhomogeneous material consisting of homogenized regions is re-homogenized by the method of a representative bulk element. As a result, transtropic effective elastic constants of composite material reinforced with a system of solid and hollow fibers is obtained.
Using the presented approach, the calculation of effective elastic constants of unidirectional composite material based on polyester resin reinforced with hollow and solid fiberglass was performed. The analysis of dependences for some effective elastic constants on the bulk content of the cavity in the fiber was conducted.

ВИЗНАЧЕННЯ ПРУЖНИХ СТАЛИХ КОМПОЗИЦІЙНОГО МАТЕРІАЛУ ІЗ СУЦІЛЬНИМИ ТА ПОРОЖНИСТИМИ ОДНАКОВОСПРЯМОВАНИМИ ВОЛОКНАМИ

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Ключові слова: *композиційний матеріал, суцільні волокна, порожнисті волокна, ефективні пружні сталі, гомогенізація, транстропний матеріал.*

У статті представлено підходи до визначення ефективних механічних характеристик композиційного матеріалу, армованого суцільними та порожнистими волокнами за допомогою методу представницького об’ємного елементу. Матеріали матриці та волокна вважалися транстропними. Взаємне розташування суцільних та порожнистих волокон є періодичним при загальній гексагональній схемі армування. Для визначення ефективних пружних характеристик композиційного матеріалу, що містить області з двома типами волокон, використано подвійну гомогенізацію. Увесь об'єм композиту розділяється на систему гексагональних областей, з яких можна виділити два типи. Перший – це суцільне волокно із оточуючою його матрицею, другий – порожнисте волокно із оточуючою його матрицею. Для попередньої гомогенізації кожного типу неоднорідної області скористаємося методом представницького об'ємного елементу. В результаті маємо гомогенізовані області, що складаються із гексагональних комірок двох типів, кожна з яких є транстропною. Площини ізотропії для обох областей співпадають. Враховуючи періодичний характер армування, гомогенізовані області із суцільними волокнами можемо представити за умовне волокно, а гомогенізовані області із порожнистими волокнами – за умовну матрицю. Маємо неоднорідний матеріал з умовними транстропними матрицею та волокном. Для неоднорідного матеріалу, що складається із гомогенізованих областей, проводимо повторну гомогенізацію методом представницького об'ємного елементу. В результаті отримаємо транстропні ефективні пружні сталі композиційного матеріалу, армованого системою суцільних та порожнистих волокон.

За допомогою представленого підходу проведено розрахунок ефективних пружних сталих однаковоспрямованого композиційного матеріалу на основі поліефірної смоли, армованого порожнистими та суцільними скловолокнами. Проведено аналіз залежностей для деяких ефективних пружних сталих від об'ємного вмісту порожнини у волокні.

Introduction. Every year the sphere of application of composite materials becomes wider, the use of composites becomes relevant even in those industries where they have not been used before. One of the most common components for the manufacture of composites is hollow fibers. Both hollow fiberglass and reinforcing elements have found their application in the manufacture of fiberglass. The study of the mechanical characteristics of fiberglass based on hollow fibers indicates the feasibility of using such composites for products under compressive loads. Thus, M.S. Aslanova, S.L. Roginsky, V.I. Dreitzer in their works concluded that the specific strength of fiberglass with hollow fibers was almost three times higher than the reinforcement of solid fibers [1, p. 183–188]. S. Kling and T. Czigany in their publication performed a comparative analysis of the use of hollow and solid glass fibers in the design of composite materials [2].

Despite the large number of studies of the stress-strain state of fibrous composites, only some theoretical studies focus on modeling the elastic behavior of composite materials with voids in the fibers [3–7]. The mechanical properties of fiberglass can be significantly influenced by changing the cross-sectional profile of the reinforcing fibers. Thus, R.V. Humphrey determined that the packaging of fiberglass with a cross section in the form of a hollow hexagon can result in obtaining a composite with a high fiber content at low weight [8, p. 401–413]. It is noted that the density of the composite based on hollow hexagonal fibers is twice lower than the density of ordinary fiberglass.

In some load schemes, the combined use of solid and hollow fibers may be appropriate, but the presence of both types of fibers in the composite leads to mathematical complications in determining the effective mechanical characteristics. To determine them, we use the approach described in [9], for a composite material reinforced with two types of unidirectional solid fibers.

Methods. Let us consider an example of such a scheme of reinforcement of a three-component composite material of solid and hollow equally directed fibers of periodic structure (Fig. 1). Let us divide the entire array of composite material by a system of hexagonal cells, as shown in Figure 1.

To determine the effective elastic constants for hexagonal cells containing hollow fibers, the formulas presented in [10] are used.

Longitudinal modulus of elasticity:

$$E_1 = E_1^* \frac{\alpha}{d^\circ \alpha - \gamma}\left(d^* f + d^\circ\left(1 - f - g\right)\right),$$

where f and g – respectively, the fiber material volume fracture and the cavity in the composite;
E_1^* – longitudinal modulus of elasticity of the matrix material;
E_2^* – the transverse modulus of elasticity of the matrix material;
ν_{12}^*, ν_{21}^*, ν_{23}^* – Poisson's ratios the matrix material;

E_1° – longitudinal modulus of elasticity of the fiber material;
E_2° – the transverse modulus of elasticity of the fiber material;
ν_{12}°, ν_{21}°, ν_{23}° – Poisson's ratios the fiber material;

$d^{\circ} = \dfrac{\alpha + \beta\nu_{12}^{\circ}}{\alpha E_1^{\circ}}$; $d^{*} = \dfrac{\alpha + \beta\nu_{12}^{*}}{\alpha E_1^{*}}$;

$\alpha = (1 - f - g)\left(\left(E_2^{*}\nu_{23}^{\circ} - E_2^{\circ}\nu_{23}^{*}\right)f - E_2^{*}(f + 2g)\right) - E_2^{\circ}(1 + f + g)f$;

$\beta = 2(f + g)\left(\nu_{21}^{\circ}E_2^{*}(1 - f - g) + \nu_{21}^{*}E_2^{\circ}f\right)$;

$\gamma = 2f(f + g)\left(\nu_{21}^{\circ}E_2^{*}\dfrac{\nu_{12}^{*}}{E_1^{*}} - \nu_{21}^{*}E_2^{\circ}\dfrac{\nu_{12}^{*}}{E_1^{\circ}}\right)$.

Poisson's ratio:

$$\nu_{12} = \frac{\nu_{21}^{*}\nu_{12}^{*}d^{\circ}\alpha - \gamma}{\nu_{21}^{*}\left(d^{\circ}\alpha - \gamma\right)}.$$

Longitudinal shear modulus:

$$G_{12} = \frac{G_{12}^{*}\left(G_{12}^{\circ}f(1 + f + g)^2 + G_{12}^{*}(f + 2g)\left(1 - (f + g)^2\right)\right)}{G_{12}^{\circ}f\left(1 - (f + g)^2\right) + G_{12}^{*}(f + 2g)(1 + f + g)^2}.$$

Transverse modulus of elasticity:
based on the equality of radial displacements

$$E_2 = \frac{2\eta E_2^{*}}{\eta\left(4\gamma_3\left(1 - \nu_{21}^{*}\nu_{12}^{*}\right) + \left(1 + \nu_{23}^{*}\right) + \gamma_1\right) + 2(f + g)\left(f\gamma_1 E_2^{\circ} - E_2^{*}\left((f + 2g)\gamma_2 + 2g\nu_{23}^{\circ}\right)\right)},$$

where

$\gamma_1 = \left(1 - \nu_{23}^{*}\right) - \dfrac{d_0 f\nu_{21}^{*}}{d^{*}f + d^{\circ}(1 - f - g)}$;

$\gamma_2 = \left(1 - \nu_{23}^{\circ}\right) - \dfrac{d^{\circ}(f + g - 1)\nu_{21}^{\circ}}{d^{*}f + d^{\circ}(1 - f - g)}$;

$\gamma_3 = \dfrac{\left((f + g)\left(3g(f + 2g) - 2(f + 3)\right) - fg\right)\chi_1 + f^2\chi_2 + \left((f + g)^2 + g(f + 2g)\right)\chi_3}{3\chi_3(g - 1)(f + 2g + 1)(f + g - 1)}$;

$\gamma_4 = \dfrac{\left((g - 1)\left(4(f + g)^2 + g(f + 2g)\right) - f^2 g\right)\chi_1 - f^2\chi_2 - \left((f + g)^2 + g(f + 2g)\right)\chi_3}{3\chi_3(g - 1)(f + 2g + 1)(f + g - 1)}$;

$d^{\circ} = \dfrac{\nu_{21}^{\circ}}{E_2^{\circ}}\left(\nu_{21}^{\circ}\left(\dfrac{\tau}{\eta}f + 1\right) - \dfrac{2\nu_{21}^{\circ}}{1 - \nu_{23}^{\circ}} - 2\nu_{21}^{*}E_2^{\circ}f\dfrac{f + g}{\eta}\right) - \dfrac{1 - \nu_{23}^{\circ} - 2\nu_{21}^{\circ}\nu_{12}^{\circ}}{E_1^{\circ}\left(1 - \nu_{23}^{\circ}\right)}$;

$d^{*} = \dfrac{\nu_{21}^{*}}{E_2^{*}}\left(\nu_{21}^{\circ}\left(\dfrac{\tau}{\eta}f + 1\right) - \dfrac{2\nu_{21}^{*}}{1 - \nu_{23}^{*}} - 2\nu_{21}^{*}E_2^{\circ}f\dfrac{f + g}{\eta}\right) - \dfrac{1 - \nu_{23}^{*} - 2\nu_{21}^{*}\nu_{12}^{*}}{E_1^{*}\left(1 - \nu_{23}^{*}\right)}$;

$$d_0 = \left(\frac{\nu_{21}^{\circ}}{E_2^{\circ}}\frac{\tau}{\eta} - 2\nu_{21}^{*}\frac{f + g}{\eta}\right)\left(\frac{E_2^{\circ}}{E_2^{*}}f\left(1 - \nu_{23}^{*}\right) - f\left(1 - \nu_{23}^{\circ}\right) - 2g\right) -$$
$$- \frac{2\nu_{21}^{*}}{E_2^{*}} + \frac{\nu_{21}^{\circ}}{E_2^{\circ}}\left(1 + \nu_{23}^{\circ}\right) + \frac{\nu_{21}^{\circ}}{E_2^{*}}\left(1 - \nu_{23}^{*}\right);$$

$$\eta = E_2^*\left(2g + f\left(1-\nu_{23}^{\circ}\right)\right)\left(f+g-1\right) - fE_2^{\circ}\left(\left(f+g\right)\left(1-\nu_{23}^*\right)+\left(1+\nu_{23}^*\right)\right);$$

$$\tau = E_2^*\left(1+\nu_{23}^{\circ}\right)\left(f+g-1\right)+E_2^{\circ}\left(\left(f+g\right)\left(1-\nu_{23}^*\right)+\left(1+\nu_{23}^*\right)\right);$$

$$\chi_1 = d_{13}t_4 - d_{23}t_3;\ \chi_2 = d_{23}t_2 - d_{13}t_1;\ \chi_3 = t_1t_3 - t_2t_4;$$

$$d_{11} = k_{11}\left(f^2+3\left(f+g\right)\left(g-1\right)\right)-3k_{12}\left(f+g\right)\left(g-1\right)\left(f+2g+1\right);$$

$$d_{12} = k_{13}\left(\left(f+g\right)^2+\left(f+g\right)\left(1-3g\right)+1\right)-3k_{14}\left(f+g\right)\left(g-1\right)\left(f+2g+1\right);$$

$$d_{13} = k_{13}E_2^*\left(f+g-1\right)\left(\left(f+g\right)^2+f+g+1\right)+$$
$$+E_2^{\circ}f\left(k_{11}\left(f^2+3g\left(f+g\right)\right)-3\left(1+\nu_{23}^*\right)\left(f+g\right)^2\left(f+g-1\right)\left(g-1\right)\left(f+2g+1\right)\right);$$

$$d_{21} = k_{21}\left(f^2+3\left(f+g\right)\left(g-1\right)\right)+3k_{22}\left(f+g\right)\left(g-1\right)\left(f+2g+1\right);$$

$$d_{22} = k_{23}\left(\left(f+g\right)^2+\left(f+g\right)\left(1-3g\right)+1\right)-3k_{24}\left(f+g\right)\left(g-1\right)\left(f+2g+1\right);$$

$$d_{23} = k_{23}E_2^*\left(f+g-1\right)\left(\left(f+g\right)^2+f+g+1\right)+$$
$$+E_2^{\circ}f\left(k_{21}\left(f^2+3g\left(f+g\right)\right)-3\left(1+\nu_{23}^*\right)\left(f+g\right)^2\left(f+g-1\right)\left(g-1\right)\left(f+2g+1\right)\right);$$

$$k_{11} = 4\left(f+g\right)^3\left(\nu_{23}^*+\nu_{21}^*\nu_{12}^*\right)-\left(1+\nu_{23}^*\right)\left(1+3\left(f+g\right)^2\right);$$

$$k_{12} = 4\left(f+g\right)\left(1-\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)\left(\left(f+g\right)^2-1\right);$$

$$k_{13} = \left(3g\left(f+g\right)^2+g^3\right)\left(1+\nu_{23}^{\circ}\right)-4\left(f+g\right)^3\left(\nu_{23}^{\circ}+\nu_{21}^{\circ}\nu_{12}^{\circ}\right);$$

$$k_{14} = \left(\left(f+g\right)^2-g^2\right)\left(1+\nu_{23}^{\circ}\right)+4g\left(f+g\right)\left(1-\nu_{21}^{\circ}\nu_{12}^{\circ}\right);$$

$$k_{21} = 2\left(f+g\right)^3\left(3+\nu_{23}^*-2\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)\left(1-3\left(f+g\right)^2\right);$$

$$k_{22} = 2\left(f+g\right)\left(\nu_{23}^*+2\nu_{21}^*\nu_{12}^*-1\right)-\left(1+\nu_{23}^*\right)\left(\left(f+g\right)^2+1\right);$$

$$k_{23} = \left(3g\left(f+g\right)^2-g^3\right)\left(1+\nu_{23}^{\circ}\right)-2\left(f+g\right)^3\left(3+\nu_{23}^{\circ}-2\nu_{21}^{\circ}\nu_{12}^{\circ}\right);$$

$$k_{24} = \left(\left(f+g\right)^2+g^2\right)\left(1+\nu_{23}^{\circ}\right)-2g\left(f+g\right)\left(\nu_{23}^{\circ}+2\nu_{21}^{\circ}\nu_{12}^{\circ}-1\right);$$

$$t_1 = d_{21}E_2^{\circ}f+k_{23}E_2^*\left(f+g-1\right)^3;\ t_2 = d_{11}E_2^{\circ}f+k_{13}E_2^*\left(f+g-1\right)^3;$$

$$t_3 = k_{11}E_2^{\circ}f^3+d_{12}E_2^*\left(f+g-1\right);\ t_4 = k_{21}E_2^{\circ}f^3+d_{22}E_2^*\left(f+g-1\right);$$

based on the equality of circumferential movements

$$E_2 = \frac{2\eta E_2^*}{\eta\left(4\gamma_4\left(1-\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)+\gamma_1\right)+2\left(f+g\right)\left(f\gamma_1E_2^{\circ}-E_2^*\left(\left(f+2g\right)\gamma_2+2g\nu_{23}^{\circ}\right)\right)}.$$

Poisson's ratio:
based on the equality of radial displacements

$$\nu_{23} = \frac{\eta\left(4\gamma_3\left(1-\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)-\gamma_1\right)-2\left(f+g\right)\left(f\gamma_1E_2^{\circ}-E_2^*\left(\left(f+2g\right)\gamma_2+2g\nu_{23}^{\circ}\right)\right)}{\eta\left(4\gamma_3\left(1-\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)+\gamma_1\right)+2\left(f+g\right)\left(f\gamma_1E_2^{\circ}-E_2^*\left(\left(f+2g\right)\gamma_2+2g\nu_{23}^{\circ}\right)\right)},$$

based on the equality of circumferential movements

$$\nu_{23}=\frac{\eta\left(4\gamma_4\left(1-\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)-\gamma_1\right)-2\left(f+g\right)\left(f\gamma_1E_2^\circ-E_2^*\left(\left(f+2g\right)\gamma_2+2g\nu_{23}^\circ\right)\right)}{\eta\left(4\gamma_4\left(1-\nu_{21}^*\nu_{12}^*\right)+\left(1+\nu_{23}^*\right)+\gamma_1\right)+2\left(f+g\right)\left(f\gamma_1E_2^\circ-E_2^*\left(\left(f+2g\right)\gamma_2+2g\nu_{23}^\circ\right)\right)}.$$

These formulas are obtained for a composite material with a transtropic matrix and a hollow fiber, if the components are isotropic, then in these formulas it is necessary to equate the corresponding elastic characteristics of the matrix and fiber materials. For hexagonal cells with solid fibers, one can use the same formulas where $g=0$.

With this partition, the matrix material in any hexagonal cell will occupy the same volume if the diameter of the fibers of both types is the same, and the volume occupied by the matrix material will be different if the diameter of the fibers of both types is different.

We have two types of hexagonal cells – with a hollow fiber (the ring is marked by an inclined hatching), and with a solid fiber (the circle is marked by an inclined hatching). Next, we approximate a hexagonal cell with a circle equal to the area of this cell. Applying the formulas of dependence of constants on the elastic characteristics of the transtropic matrix, fiber and bulk particles of the fiber and cavity material in the composite material, we determine the elastic constants for the first area of the composite and, taking in the formulas $g=0$, determine the elastic constants for the second area of the composite. As a result, we obtain a two-component fibrous material with calculated elastic constants of transversely isotropic matrix and fibers (Fig. 1).

For the obtained composite material, we determine the elastic constants using the method described above. Then the section of this material is divided into hexagonal cells, and we apply to the elementary hexagonal cell the procedure for determining the elastic constants, according to the previously defined elastic constants of the matrix and fibers. The volume fraction of a fiber is defined as the ratio of the area of the circle occupying the fiber to the area of the hexagonal cell. The obtained elastic constants of the composite material will determine the elastic constants of the three-component composite material with two types of fibers.

If in the original three-component composite material the volume fraction of the material of the hollow fiber of the grade f_1 with the cavity g, and the solid fiber – f_2, the volume fraction of the matrix material will be $1-f_1-f_2-g$. Then, according to the above approach to the representation of the matrix and fiber, the volume fraction of the new matrix will be $\left(1-f_1-f_2-g\right)\dfrac{f_1}{f_1+f_2+g}+f_1$. Thus, the volume fraction of the new matrix in the two-component fibrous material will be equal to $\dfrac{f_1}{f_1+f_2+g}$. Similarly, let us determine the volume fraction of new fiber in a two-component fibrous material. The volume fraction of the fiber will be the value, which is equal to $\left(1-f_1-f_2-g\right)\dfrac{f_2}{f_1+f_2+g_2}+f_2$. Thus, the volume fraction of fiber in the composite material will be the value $\dfrac{f_2}{f_1+f_2+g}$.

However, it should be taken into account that the obtained ratios for calculating the elastic properties of the composite material operate only with the volume fraction of fibers and do not take into account the

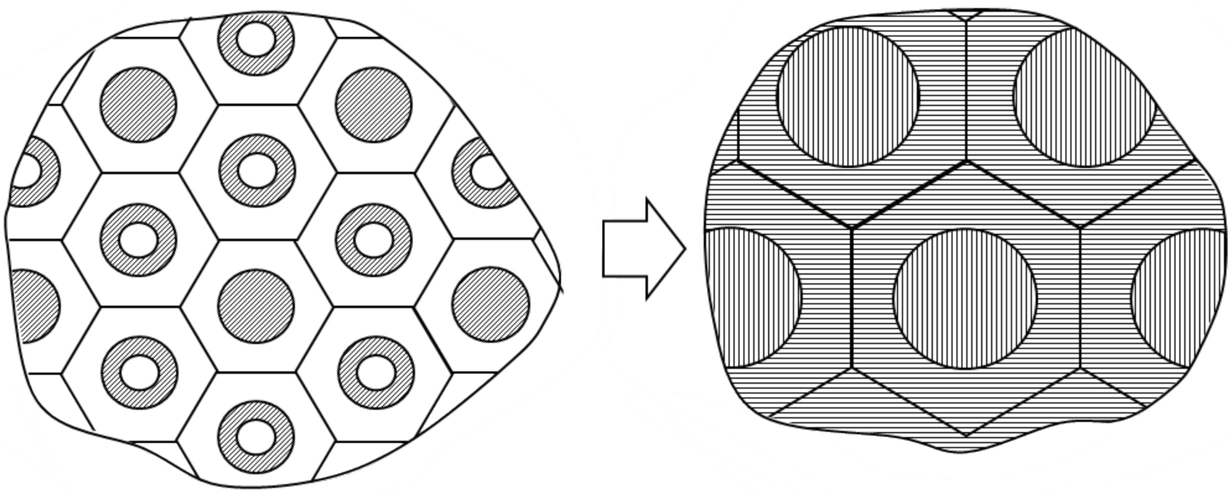

Fig. 1. Representation of a three-component composite material

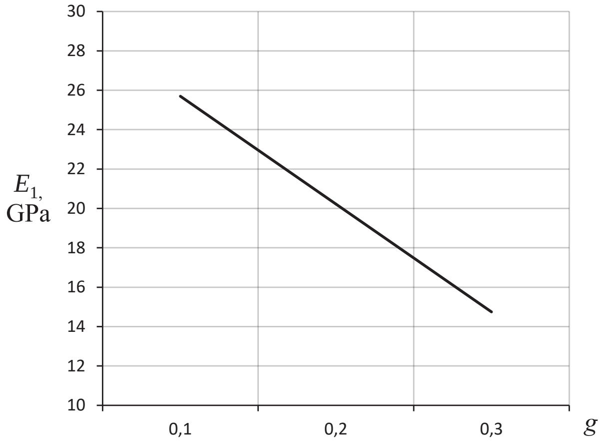

Fig. 2. Effective longitudinal modulus of elasticity of a composite material reinforced with solid and hollow fibers

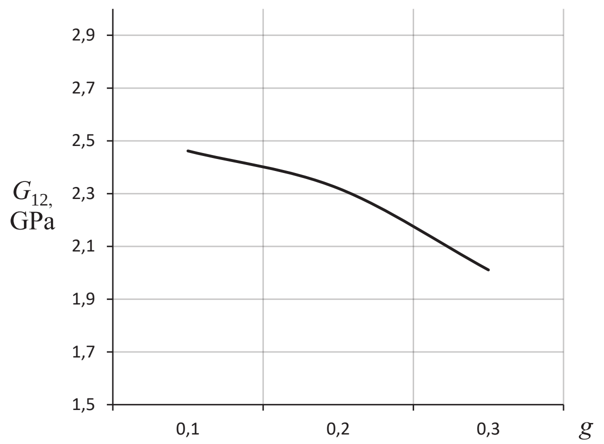

Fig. 3. Effective shear modulus of a composite material reinforced with solid and hollow fibers

diameter of the fiber and the structure of the stacking. Therefore, the above technique can be applied to other schemes of reinforcement, it should only be taken into account that the more precisely the circle approximates the boundary of the matrix material in the model, the more accurate the results are.

Numerical calculation. Let us consider a unidirectional UD GFRP composite based on polyester resin (Polimal 109), reinforced with fiberglass (E-glass). The elastic characteristics of the components [11]: for fiber we have $E^{\circ} = 73$ GPa, $\nu^{\circ} = 0,22$, $G^{\circ} = 29,9$ GPa; for matrix $-E^{*} = 3,24$ GPa, $\nu^{*} = 0,385$. Suppose that the scheme of reinforcement is the same as Fig. 1. Volumetric content of hollow fiber $f + g = 0,4$.

Figures 2 and 3 show the dependence of the effective longitudinal modulus of elasticity and shear modulus on the volumetric content of the cavity.

Conclusions. The method of double homogenization for a composite material periodically reinforced with two types of fibers – solid and hollow, is proposed. Presented this technique for calculating the effective elastic characteristics of the composite material on the example of a composite with isotropic properties of the components. It can be noted that the increase in the volume content of the cavity leads to a decrease in the values of the effective characteristics, and for an effective longitudinal modulus of elasticity, this dependence is linear, and for an effective shear modulus it is nonlinear.

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