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TIMOSHENKO EQUATION, VIOLATION OF CONTINUITY AND SOME APPLICATIONS

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the Timoshenko equation, hyperbolicity, violation of continuity, elastic foundation, wavelength, frequency, Euclidean space.

We study the Timoshenko model of bending beam vibrations, that includes the beginning from a brief general consideration and the fast transition from n-dimensional Euclidean space to 4-dimensional space with respect to spatial coordinates and time. As a result, the Timoshenko equation is obtained on the basis of a mathematical approach, without a correction coefficient (shear coefficient) as a special case of a more general our extended refined equation. We investigate the problem of the effect of liquid, as a special case of an elastic base, on shear in Timoshenko elastic plate. It is shown that any media contacting with the plate reduce the shear effect. The violation of continuity is noted, which has not been considered previously. The works based on the Timoshenko model are presented for a beam on an elastic base. In the case of a rectangular change in the cross section, another matching problem immediately arises, connected with appearing reflected and transmitted waves. From the solvability of the problem for the phase velocity in the case of short wavelengths (high frequencies), the yield to the characteristic is studied and it is shown that in connection with the violation of continuity, the applicability of the classical theory takes place at wavelengths of more than 5 thicknesses. The problem of elastic plates floating on a liquid layer is studied in detail, using various theories. Variational formulations without taking into account the violation of continuity are considered and commented, the separation of variables in the Timoshenko equation is considered.

РІВНЯННЯ ТИМОШЕНКА, ПОРУШЕННЯ БЕЗПЕРЕРВНОСТІ І ДЕЯКІ ЗАСТОСУВАННЯ

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Ключові слова:
рівняння Тимошенка, гіперболічність, порушення суцільності, пружна основа, довжина хвилі, частота, евклідовий простір

Досліджено модель Тимошенка згинних коливань балки, що включає спочатку загальні міркування і перехід від n-мірного евклідова простору R^n до 4-мірного простору відносно просторових координат x_1, x_2, x_3 і часу t. На основі математичного підходу рівняння Тимошенка одержано без коректуючого коефіцієнта (коефіцієнт зсуву) як окремий випадок більш загального розширеного рівняння. Досліджено задачу впливу рідини як окремий випадок пружної основи в пластині Тимошенка. Показано, що будь-яке середовище, що контактує з пластиною нівелює ефект зсуву. Відмічено порушення суцільності, яке раніше не розглядалося. Дослідження, основані на моделі Тимошенка, представлялись для балки на пружній основі. У випадку прямокутного виду поперечного перерізу виникає інша задача, пов’язана з появою відбитих хвиль. З розв’язку задачі досліджується фазова швидкість у випадку коротких довжин хвиль (високі частоти), виявлено, що у випадку порушення суцільності застосування класичної теорії обмежено довжиною хвиль, що перевищує 5 товщин балки. На основі різних теорій детально вивчено задачу про пружні пластини, що плавають на рідкому шарі. Описуються і обговорюються варіаційні формулювання без урахування порушення суцільності, розглядається відокремлення змінних в рівнянні Тимошенка.

1. Introduction

The Cauchy-Poisson method was proposed considering the bending vibrations of an elastic beam-plate based on the equations of elastodynamics (Cauchy, 1828) [1], (Poisson, 1829) [2]. A generalization of the Cauchy-Poisson method to n-dimensional Eucledian space was obtained in (Selezov, 2000) [3]. The violation of continuity was shown in (Selezov, 2018) [4].

The violation of continuity in the Timoshenko model has been not investigated in all previous considerations and it is an absolutely new problem under consideration. That is why, a numerous of traditional investigations of the Timoshenko equation is not considered here. Moreover, these investigations are else considered in detail in a book dedicated to the Timoshenko (P: Statement of the problem Grigolyuk and Selezov, 1973) [5].

The paper consists of some points presented below. Statement of the problem in Eucledian space presents the problem in Eucledian space R^n represented by a finite system of partial differential equations. Corresponding boundary-value problem and some assamptions are considered; Extended refined equation in 4-th dimensional space presents the problem in 4-dimensional Eucledian space and an extended generalized refined equation including the Timoshenko equation as a partcular case; Violation of continuity and the effect of elastic foundation are noted and commented and the influence of the elasticity of the base on processes is investigated*;* Wave propagation in elastic floating plate presents the problem of wave propagation in a floating elastic plate; On variational formulations without violation of continuity considers variational principles without the violation of continuity and an asymptotic approach which are based on the continuity of elastic media; Separation of variables in the Timoshenko equation shows the application of the method of separation of variables.

2. Statement of the problem in euclidean space

We consider in Eucledian space R^n with coordinates x^q, $q=\overline{1,\, n}$ a mathematical model represented by a finite system of partial differential equations for which a boundary-value problem is posed in a domain $\Omega\times\left[0, X^m\right]$, $X^m>0$ bounded by hypersurfaces (the index is fixed):

$$\Omega=\begin{cases} x\in R^n : -\infty < \left(x^1, x^2, \ldots, x^{s-1}, x^{s+1}, \ldots, x^{n-1}\right) < \infty, \\ x^n \ \geq\ 0, \quad -h^s \leq x^s \leq\ h^s. \end{cases}$$

We assume that hypersurfaces $x^s=\pm h^s$ are removed from the middle hypersurface $x^s=0$ and it is considered the composition with respect to $x^s=0$. The case is considered when conditions are given on hypersurfaces $x^s=\pm h^s$.

It is assumed that the model depends on a finite number ν of parameters ε_r, $r=\overline{1,\nu}$. Formally, such a model can be defined as a system k of differential equations in partial derivatives of p-th order with k unknowns u_i $\left(i=\overline{1,k}\right)$ and n arguments (Dunford & Schwartz,
1969) [6]

$$F_i\left(x^1,\ldots,x^n;u_1,\ldots,u_k;u_{1,1},\ldots,u_{k,n};\ldots u_{\underset{P\ times}{1,\ 1\ldots1}},\ldots,u_{\underset{P\ times}{k,\ n\ldots n}};\varepsilon_1,\ldots,\varepsilon_\nu\right)=P_i\left(x^1,\ldots,\ x^n\right), \tag{1}$$

$$j=\overline{1,(k\cdot p)} \text{ in } \Omega.$$

The following system of boundary conditions on hypersurfaces $x^s=-h^s$, $x^s=h^s$ is defined

$$f_j\left(x^1,\ldots,x^n;u_1,\ldots,u_k;u_{1,1},\ldots,u_{\underset{(P-1)\ times}{k,\ \ n\ldots n}};\varepsilon_1,\ldots,\varepsilon_\nu\right)\Bigg|_{x^s\ =\ \pm\ h^s}=Q_j^{\pm}, \quad j=\overline{1,(k\cdot p)}. \tag{2}$$

Here, the index after the “comma” denotes differentiation with respect to the corresponding coordinate, in the general case $p \neq n$ it depends on all possible partial derivatives up to the p -th order inclusively, the position of the hypersurface may depend on u_i and their derivatives. The solution of the boundary value problem (l), (2) consists of determination of the functions u_i transforming equations (1) into identities, and in selection of a set of these functions those functions that satisfy conditions (2).

3. Extended refined equation in 4- th dimensional space

Further we consider 4-th dimensional space with respect to spatial coordinates x_1, x_2, x_3 and time t. When constructing a generalized equation, dimensionless quantities are introduced, taking thickness $2h$ (m), shear wave velocity c_s (m / s), and elastic medium density ρ (kg / m) as characteristic quantities

$$u_k^* = \frac{1}{2h} u_k, \quad \left(x_1^*, x_2^*\right) = \frac{1}{2h}\left(x_1, x_2\right),$$

$$t^* = \frac{c_s}{2h} t, \quad q^* = \frac{1}{G} q,$$

$$h^* = \frac{1}{2}, \quad c_s^2 = \frac{G}{\rho}.$$

In the study of wave propagation, dimensionless quantities are introduced: $l^* = \frac{1}{2h} l$ is the wavelength, $c^* = \frac{c}{c_s}$ is the phase velocity.

The extended differential equation for the transverse coordinate $u_3 = w_0$ has the form (asterisks are omitted)

$$\left\{\left[\left(\frac{\partial^2}{\partial t^2} + a_1 \nabla^2 \nabla^2\right)_K - a_2 \frac{\partial^2}{\partial t^2} \nabla^2 + a_3 \frac{\partial^4}{\partial t^4}\right]_{TM} - b_1 \nabla^2 \nabla^2 \nabla^2 + b_2 \frac{\partial^2}{\partial t^2} \nabla^2 \nabla^2 - b_3 \frac{\partial^4}{\partial t^4} \nabla^2 + b_4 \frac{\partial^6}{\partial t^6}\right\}_{TMC},$$

$$w_0 = \left\{\left[1 - d_1 \nabla^2 + d_2 \frac{\partial^2}{\partial t^2}\right]_{TM} + d_3 \nabla^2 \nabla^2 - d_4 \frac{\partial^2}{\partial t^2} \nabla^2 + d_5 \frac{\partial^4}{\partial t^4}\right\}_{TMC} \left(q^+ - q^-\right). \quad (3)$$

In (3), the following notations are accepted: $w_0(x_1, t)$ transverse deviation (deflection), t is a time, $(q_1 - q_2)$ transverse load,

An operator with index K corresponds to the Bernoulli-Euler equation (extended to plates by Kirchhoff). The operator with the TM index corresponds to the Timoshenko equation (extended to plates by Ufland and developed by Mindlin). The Rayleigh equation is included in the operator TM with $a_3 = 0$. An operator with the TMC index corresponds to the extended equation (constructed by Selezov). It follows from the above analytical construction, as a special case, the Timoshenko equation, but without the introduction of a correction parameter (the shear coefficient).

The Timoshenko equation is of hyperbolic type as a generalization of 4-th order parabolic equation, rather than 2-th order equation, which only in this case has always been generalized before.

With increasing frequencies. those. as the wavelength decreases and the characteristic is reached, violation of continuity occurs in accordance with the Timoshenko model.

When deriving the Timoshenko equation, the slope of the tangent to the bend curve is postulated. those it is represented in the form $\partial w / \partial x = \psi + \gamma$ where ψ is the bending deformation, γ is the shear deformation. At high frequencies and sharp inhomogeneities, this will manifest itself.

4. Violation continuity and the effect of elastic foundation

From the analysis (Selezov, 2018) [4], it was found that the Timoshenko model is applicable at wavelengths λ of more than five thicknesses h, that is, $\frac{\lambda}{h} > 5$ when the influence of the thickness shear is already small and there is no discontinuity. We considered a beam-strip of an elastic plate, for which they were derived strictly mathematically, following Cauchy and

Poisson. refined equations, including the Timoshenko equation as a special case.

The effect of an elastic base was investigated in (Selezov and Korsunsky, 1991) [7], in which it was shown that this reduces the effect of thickness shear in the Timoshenko model. Note that water can also be considered as an elastic base.

For the first time, a beam on an elastic foundation was examined by Timoshenko (Timoshenko, 1956) [8]. After his emigration to America, works appeared on the effect of an elastic foundation on the shear deformation in his equation. For example, in (Achenbach et al., 1967) [9], the propagation of free elastic waves in a plate lying on an elastic half-space was studied. It was shown in (Yu, 1960) [10] that in a three-layer plate, the effect of shear and inertia of rotation of the outer plates relative to their middle surfaces is negligible. In (Lloid and Miklowitz, 1962) [11], vibrations of an elastic plate on an elastic base are considered.

In well-known works considering the Timoshenko beam of variable cross-section, no conditions were imposed on the value of the permissible change in the cross-section, which can lead to the violation of continuity. we present only one of them (Shubov, 2002) [12].

5. Wave propagation in elastic floating plate

We consider the problem of the propagation of plane unsteady bending waves in elastic plate located on a liquid surface of finite depth d, assuming that in time $t=0$ a stationary normal load at a point is applied to the surface $p = p_0 p_1(x) p_2(t)$. The plate bending motions are described by a refined theory, taking into account the inertia of rotation and transverse shear deformation (Grigulyuk and Selezov, 1973) [5], and the fluid is considered compressible isentropic. The corresponding initial-boundary-value problem is formulated to find solutions to the system

$$D\psi_{xx} - k^2 Gh(\psi + w_x) = \rho_1 I \psi_{tt}, \qquad (4)$$

$$k^2 Gh(w_{xx} + \psi_x) - \\ -\rho_2 w - \rho_2 \varphi_t|_{x=0} + p = \rho_1 h w_{tt}, \qquad (5)$$

$$\varphi_{xx} + \varphi_{zz} - c_0^{-2}\varphi_{tt} = 0, \qquad (6)$$

$$\rho_2 x \in (-\infty, \infty); \; z \in [-d, 0]; \; t \in [0, \infty).$$

Under boundary conditions

$$w_t = \varphi_z|_{z=0}; \; \varphi_z|_{z=-d} = 0 \qquad (7)$$

and initial conditions for $t=0$

$$\psi = 0; \quad \psi_t = 0; \quad w = 0; \\ w_t = 0; \quad \varphi = 0; \quad \varphi_t = 0. \qquad (8)$$

The boundary conditions (7) express the equality of the vertical velocity components of the plate and the liquid at the interface and the impermeability condition at the bottom. Value $\psi(x, t)$ is the angle of rotation of the plate; φ is the potential of fluid velocities, ρ_1 and ρ_2 are the densities of the plate and fluid, $k^2 = 2\big/\left(2 - \nu + \sqrt{0,5-\nu}\right)$ is the shear coefficient, $I = h^3/12$ moment of inertia of the cross section, c_0 the speed of sound and fluid. In (4) (8) and further, dimensionless quantities are introduced by the formulas

$$\left(x^*, z^*, w^*, d^*\right) = \frac{1}{h}(x, z, w, h, d),$$

$$t^* = \frac{\sqrt{gh}}{h} t, \quad P_0^* = \frac{p_0}{gh\rho_1},$$

$$c_0^* = \frac{c_0}{\sqrt{gh}}, \quad \varphi^* = \frac{\varphi}{h\sqrt{gh}},$$

$$l^* = \frac{I}{h^3}, \quad D^* = \frac{D}{gh^4\rho_1}, \quad G^* = \frac{g}{gh\rho_1},$$

$$\lambda^* = \frac{\lambda}{gh\rho_1}, \quad \rho_i^* = \frac{\rho}{\rho_i}(i = 1, 2).$$

The above statement (4)-(8) also includes special cases. So, for an incompressible fluid ($c_0 \to \infty$), instead of the wave equation (6), the Laplace equation $\varphi_{xx} + \varphi_{zz} = 0$ is solved; plate motion is described by the classical Kirchhoff theory:

$$Dw_{xxxx} + \rho_1 w_{tt} + \rho_2 w + \rho_2 \varphi_t|_{z=0} = p; \qquad (9)$$

the movement of the plate is described by the equation taking into account only the inertia of rotation:

$$Dw_{xxxx} - \rho_1 I w_{ttxx} + \\ +\rho_1 h w_{tt} + \rho_2 w + \rho_2 \varphi_t|_{z=0} = p, \qquad (10) \\ \psi + w_x = 0;$$

the movement of the plate is described by the equation, taking into account only the transverse shear strain

$$Dw_{xxxx} - \frac{\rho_1 D}{k^2 G} w_{ttxx} + \rho_1 h w_{tt} +$$
$$+\left(1 - \frac{D}{k^2 Gh}\partial_{xx}\right)\left(\rho_2 w + \rho_2 \varphi_t\big|_{z=0} - p\right) = 0; \quad (11)$$
$$D\psi_{xx} - k^2 Gh(\psi + w_x) = 0.$$

To solve this problem, the integral Fourier transforms in coordinate x and Laplace transforms in time t are applied:

$$F(\kappa) = \int_{-\infty}^{\infty} f(x) e^{-i\kappa x} dx;$$
$$\overline{f}(s) = \int_{0}^{\infty} f(t) e^{-st} dt.$$

The transition from the space of Laplace images to the space of originals in some cases is carried out by (Deutsch, 1956) [13]. In other cases, the original is found by casting an integral of the Riemann-Mellin type.

$$f(t) = \frac{1}{2\pi i}\int_{c-i^{\infty}}^{c+i^{\infty}} \overline{f}(s) e^{st} ds,$$

to the Fourier transform (Krylov and Skoblya, 1974) [14]

$$f(t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty}\left[\mathrm{Re}\left(\overline{f}(s)\right) + \mathrm{Im}\left(\overline{f}(s)\right)\right] e^{i\tau t} d\tau,$$

where $s = c + i\tau$.

In the general case, the solution of the problem (4)-(11) under consideration has the form

$$w = \frac{p_0}{4\pi^2 i}\int_{c-i_\infty}^{c+i_\infty}\int_{-\infty}^{\infty}\frac{P_1(\kappa)\overline{p}_2(s)\eta_0\kappa_1 e^{i\kappa x+st} d\kappa ds}{\kappa_2\left(C_1 s^4 + 2C_2 s^2 + C_3^2\right)}. \quad (12)$$

The choice of other parameters is determined by the model.

So, for the Kirchhoff model (9) we have

$$\kappa_1 = 1, \quad \kappa_2 = \rho_1 h\eta_0 + \rho_2 cth\eta_0 d,$$
$$C_1 = 0, \; C_2 = 1/2, \; C_3^2 = \eta_0\left(D\kappa^4 + \rho_2\right)/\kappa_2.$$

In the case of the Timoshenko plate (4), (5)

$$\kappa_1 = 1 + \frac{\kappa^2 D}{k^2 Gh} + \frac{s^2 \rho_1 I}{k^2 Gh},$$
$$\kappa_2 = \frac{\rho_2 I}{k^2 G}\left(\eta_0 + \frac{\rho_1}{h} cth\eta_0 d\right); \; C_1 = 1;$$

$$C_2 = \left\{\left[\rho_1 h\left(1 + \frac{\kappa^2 D}{k^2 Gh}\right) + \rho_1 I\left(\frac{\rho_2}{k^2 Gh} + \kappa^2\right)\right]\eta_0 + \right.$$
$$\left. + \rho_2\left(1 + \frac{\kappa^2 D}{k^2 Gh}\right) cth\eta_0 d\right\} \Big/ 2\kappa_2;$$
$$C_3 = \eta_0\left[D\kappa^4 + \rho_2\left(1 + \frac{D\kappa^2}{k^2 Gh}\right)\right] \Big/ \kappa_2.$$

For the inertia of rotation model

$$\kappa_1 = 1; \; \kappa_2 = \left(\rho_1 I\kappa^2 + \rho_1 h\right)\eta_0 + \rho_2 cth\eta_0 d;$$
$$C_1 = 0, \; C_2 = 1/2, \; C_3^2 = \eta_0\left(D\kappa^4 + \rho_2\right)/\kappa_2.$$

For the transverse shear strain model (11)

$$\kappa_1 = 1 + \frac{D\kappa^2}{k^2 Gh};$$
$$\kappa_2 = \left(1 + \frac{D\kappa^2}{k^2 Gh}\right)\left(\rho_1 h\eta_0 + \rho_2 cth\eta_0 d\right),$$
$$C_1 = 0, \; C_2 = 1/2,$$
$$C_3^2 = \eta_0\left[D\kappa^4 + \rho_2\left(1 + \frac{D\kappa^2}{k^2 Gh} + \rho_2\right)\right] \Big/ \kappa_2.$$

The normal component of the tensor is

$$\sigma_{xx} = (\lambda + 2G)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z}, \quad (13)$$

where the components u and w of the displacement vector has the form (Grigolyuk and Selezov, 1973) [5]: for the Kirchhoff model $u = -z\frac{\partial w}{\partial x}$, and in other cases $u = z\psi$. Then expression (13), taking into account (12), is reduced to the form

$$\sigma_{xx} = \frac{zp_0(\lambda + 2G)}{4\pi^2 i} \times$$
$$\times\int_{c-i_\infty}^{c+i_\infty}\int_{-\infty}^{\infty}\frac{P_1(\kappa)P_2(s)\eta_0\kappa^2 e^{i\kappa x+st} d\kappa ds}{\kappa_2\left(C_1 s^4 + 2C_2 s^2 + C_3^2\right)}. \quad (14)$$

For an incompressible fluid, the transition from the space of Laplace images to the space of originals is carried out according to (Deutsch, 1956) [13] based on the convolution theorem. In this case, we have the following expressions:

$$\frac{\bar{p}_2(s)}{s^2+C_3^2} \to \frac{1}{C_3}\int\limits_0^t p_2(t-\tau)\sin C_3\tau d\tau\,, \tag{15}$$

$$\frac{\bar{p}_2(s)}{s^4+2C_2s^2+C_3^2} \to \begin{cases} 0{,}5C_2^{-3/2}\int\limits_0^t p_2(t-\tau)\left(\sin C_2^{1/2}\tau - C_2^{1/2}\tau\cos C_2^{1/2}\tau\right)d\tau, \\ 0{,}25C_2^{-3/2}\left(C_2^2-C_3^2\right)^{-1/2}\int\limits_0^t p_2(t-\tau)\left(s_1\sin s_2\tau - s_2\sin s_1\tau\right)d\tau, \\ C_2^2-C_3^2>0, \end{cases}$$

where $s_{1,2}^2 = C_2 \pm \sqrt{C_2^2-C_3^2}$.

Let us consider the case when the load in spatial and temporal coordinates changes according to the laws

$$p_1(x) = x_0^2\big/\left(x_0^2+x^2\right);$$

$$p_2(t) = \left\{t/a,\ 0\le t\le a;\ e^{-b(t-a)},\ t>a\right\}.$$

Then, for an incompressible fluid, from (14), taking into account (15), we obtain the expression of the normal component of the stress tensor in particular cases (9)-(11)

$$\sigma_{xx} = \frac{zp_0(\lambda+2G)}{\pi}\int\limits_0^\infty \frac{\kappa^3P_1(\kappa)f(t)}{\kappa_2}\cos\kappa x\,d\kappa\,,$$

where $P_1(\kappa)=\kappa_0e^{-x_0\kappa}$;

$$f(t) = \frac{1}{C_3^2a}\left(t-\frac{\sin C_3t}{C_3}\right)+$$
$$+f_1(t);\ \ f_1(t)=0;\ t\le a,$$
$$f_1(t) = -\frac{1}{C_3^2a}\left(t_1-\frac{\sin C_3t_1}{C_3}\right)+$$
$$+\frac{1}{C_3^2\left(b^2+C_3^2\right)}\left(b\sin C_3t_1 - C_3\cos C_3t_1\right)+$$
$$+\frac{e^{-bt_1}}{C_3\left(b^2+C_3^2\right)}-\frac{1-\cos C_3t_1}{C_3^2},\ \ t_1=t-a>0.$$

Numerical calculations were performed at the normal stress σ_{xx} following parameter values: $\rho_1 = 910$ kg/m³; $\rho_2 = 1000$ kg/m³; $c_0 = 1400$ m/s; $\nu = 0{,}33$; $E = 5{,}88\cdot10^9$ N/m²; $d = 20$ m; $h = 1$ m; $\kappa_0 = 1$; $a = 10^{-1}$; $b = 10^4$; $z = h/2$.

Comparison of normal stress calculations σ_{xx} is performed in cases where the plate is described by the Kirchhoff model with and without liquid. It is shown that taking the liquid into account leads to a significant decrease of the normal stress in the plate. Accounting for shear deformation significantly reduces the value σ_{xx} and in addition, leads to a shift in the maximum values.

6. On variational formulations without violation of continuity

In most studies, variational formulations and asymptotic approaches of the Timoshenko model using the law of continuity of the medium show the incorrectness of the Timoshenko model. Therefore, all further arguments and conclusions about the frequency spectra and the meaning of the second spectrum remain in question (Barbashov & Nesterenko, 1983) [15], (Nesterenko, 1989) [16], (Nesterenko, 1993) [17], (Chervyakov & Nesterenko, 1993) [18].

An attempt to use the mathematical method of asymptotic expansions taking into account the continuity of the medium leads to the incorrectness of the Timoshenko equation (Bakhvalov and Eglit, 2005) [19].

7. Separation of variables in the timoshenko equation

We also note the fundamental difference between the Rayleigh equation

$$\xi\frac{\partial^2 w}{\partial t^2}+\xi^3a_1\frac{\partial^4 w}{\partial x^4}-$$
$$-\xi^3a_2\frac{\partial^4 w}{\partial t^2\partial x^2}=\left(q^+-q^-\right), \tag{16}$$

including the Euler-Bernoulli equation, and the Timoshenko equation

$$\xi \frac{\partial^2 w}{\partial t^2} + \xi^3 a_1 \frac{\partial^4 w}{\partial x^4} - $$
$$-\xi^3 a_2 \frac{\partial^4 w}{\partial t^2 \partial x^2} + \xi^3 a_3 \frac{\partial^4 w}{\partial t^4} = \qquad (17)$$
$$= \left(1 - \xi^2 d_1 \frac{\partial^2}{\partial x^2} + \xi^2 d_2 \frac{\partial^2}{\partial t^2}\right)\left(q^+ - q^-\right).$$

A classic method of the variable separation

$$w(x,t) = W(x)T(t),$$

does not lead to the separation of variables in equation (17), in contrast to the complete separation of variables in equation (16). In the case of harmonic oscillations, the method is applicable to (16) and (17).

8. Conclusion

The violation of continuity in the Timoshenko beam equation at short wavelengths has been shown and discussed. It follows from the generalized refined equation of 4-dimensional Eucledian space obtained in this paper as a special case. It was considered the conclusions about the incorrectness and inconsistency of the Timoshenko model in well-known variation formulations and asymptotic approaches based on the law of continuity. A decrease in the influence of shear deformation in the Timoshenko equation was noted upon contact of the beam-strip with an elastic base and water, as well as a special case of an elastic base. It was shown when the violation of continuity at high frequencies and sharp changes in the beam thickness the Timoshenko model is not applicable. A decrease of the shear effect in the Timoshenko equation was shown from the solution of the problem for a floating elastic plate using the classical Kirchhoff equation and the refined Timoshenko equation. It was noted the inapplicability of separation of variables in the Timoshenko equation.

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