

# РОЗДІЛ І. ПРИКЛАДНА МАТЕМАТИКА

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## THE EFFECT OF THE VARIABILITY OF EXTERNAL PRESSURE ON THE LOCAL AND OVERALL BUCKLING LOADS FOR THE REINFORCED COMPOUND TYPE “BARREL-OGIVE” SHELL STRUCTURE WITH COMPARTMENTS OF DIFFERENT GAUSSIAN CURVATURES

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**Key words:** *buckling, critical loads, “barrel-ogive” shell design, a sign of Gaussian curvature of the middle surface, rational rigidity of circular reinforcements.*

The problem of the stability of the compound shell design with the middle surface meridian of various shapes under the simultaneous effect of variable distributed external pressure and axial stretching-compression is analyzed using numerical approach. The “barrel-ogive” type structures introduced in the previous papers is generalized in the case of the presence of a negative Gaussian curvature sign on one of its compartments. The impact on the reinforced by discretely distributed circular ribs compound system under the external pressure of various configurations is studied.

The solution is based on the use of the finite differences method with respect to the resolution equations for each compartment under the conditions of

interfacing sections through the docking rib. The impact on the stability of the of the sign Gaussian curvature of the system under study, the laws of the external pressure distribution on the longitudinal coordinate and axial loads, taking into account the stiffness of the intermediate rib from the plane of the initial curvature are analyzed. Particular attention is paid to the analysis of the external load influence on the effects of the interaction between local and overall buckling modes with the determination of the stability of the compound shell structure under combined load. The proposed algorithm for visualization of the deformation character for the shell design allows for local and general buckling mode analysis, compound shell structure “barrel-ogive” generalized type structures, to conclude its rational characteristics with identifying potentially hazardous areas of destruction.

## **ЕФЕКТ ВПЛИВУ ЗМІННОСТІ ЗОВНІШНЬОГО ТИСКУ НА ЛОКАЛЬНІ І ЗАГАЛЬНІ КРИТИЧНІ НАВАНТАЖЕННЯ ПІДКРІПЛЕНОЇ СКЛАДЕНОЇ ТИПУ «БОЧКА-ОЖИВАЛО» ОБОЛОНКОВОЇ КОНСТРУКЦІЇ З ВІДСІКАМИ РІЗНОЇ ГАУСОВОЇ КРИВИЗНИ**

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**Ключові слова:** *випинання, критичні навантаження, 'бочка-оживало' оболонкова конструкція, знак Гаусової кривизни серединної поверхні, раціональна жорсткість кругових підкріплень.*

На основі запропонованого чисельного підходу надається аналіз стійкості складеної оболонкової конструкції з меридіаном серединної поверхні різної форми, що знаходиться під одночасною дією нерівномірно розподіленого зовнішнього тиску і осевого розтягування-стискання. Введений у попередніх роботах вид конструкцій типу 'бочка-оживало' узагальнений на випадок наявності від'ємного знаку Гаусової кривизни на одному з її відсіків. Вивчена дія на підкріплену

дискретно розподіленими шпангоутами оболонкову систему нерівномірного зовнішнього тиску при наявності осьових зусиль.

Розв'язання базується на застосуванні методу скінченних різниць по відношенню до основних диференціальних рівнянь деформування для кожного відсіку за умов сполучення секцій через стикувальний шпангоут. Досліджується вплив на стійкість конструкції параметрів Гаусової кривизни серединної поверхні досліджуваної системи, законів розподілу зовнішнього тиску за повздовжньою координатою і осьових зусиль з урахуванням жорсткості проміжного шпангоута з площини початкової кривизни.

Особлива увага зосереджена на аналізі впливу характеру зовнішнього навантаження на ефекти взаємодії локальних і загальних форм випинання з визначенням стійкості складеної оболонкової конструкції при комбінованому навантаженні.

Запропонований алгоритм візуалізації характеру деформації оболонкової конструкції дозволяє провести аналіз локальних і загальних форм випинання, зробити висновок про її раціональні характеристики з точки зору стійкості складеної оболонкової конструкції узагальненого типу 'бочка-оживало' з виявленням потенційно небезпечних зон руйнування.

## Introduction

The stability study of thin shell designs is dictated by the needs and development of modern engineering, instrument making, chemical industry, aviation, rocket and space technology. The choice of effective forms of shell structures and reinforcing elements depends on specific operating conditions, in particular, on the nature of aerodynamic loading, combining external loads [1; 2] and represents a relevant problem when choosing rational geometric and rigidity characteristics of the designed systems for new techniques. A significant part of theoretical and experimental studies is aimed at compound shell structures, 'cylinder-cone' type, of zero Gaussian curvature for the middle surface [3; 4].

The initial imperfections of the middle surface, or its deviations from classical one, can lead to significant influence on the values of critical loads [5–10]. Thus, in the papers [5; 6] the effect of sign Gaussian curvature of the middle surface on the stability of individual sections of the compound shell structure and axial loading on the value of critical pressure are investigated. To buckling problem of the shell structures for negative Gaussian curvature are devoted [9; 10].

As the analysis of the current state of the problem under study shows, the variability of external pressure on the coordinate may in some cases significantly affect for stability of the shell design [11–14]. The influence of discretely located circular ribs on the stability of the shell structures is considered in [4; 5; 6], where, in particular, the problem of rational choice of their rigidity parameters was investigated.

It should be noted that the analysis of the buckling modes of the compound shell design, the detection of the zones of the largest deflection are contributed to an algorithms of computer visualization of the release process, which is dedicated to monograph [14].

The purpose of this paper is to study the effects of the variability influence of external pressure on the local and overall critical loads of the reinforced compound type "barrel-ogive" shell structure, taking

into account axial loading and the sign of the Gaussian curvature of the middle surface compartments with visualizing the buckling modes structure under combined loading.

### 1. Setting the problem and resolution equations.

Following [5; 6], a compound shell construction consisting of sections in which the deviation from the ideal shape of the forming (cylinder and cone) has a sinusoidal shape. Holding to the terminology introduced in these papers in the case of convexity of the compartments, which corresponds to the positive direction of the Gaussian curvature of the middle surface, will be called the shell of the 'barrel' or 'ogive' compartments, respectively. In the case of negative curvature of the meridian, the terms with the prefix 'pseudo' ('pseudo-barrel' or 'pseudo-ogive') are introduced. The compound shell structure of a constant thickness  $h$ , a module of the elasticity  $E$  of the material and the coefficient of Poisson  $\nu$  is investigated. In accordance with the theory of gentle shells of 'medium length' [15; 16], a restriction is imposed on the geometry of the design, according to which the relative height of deviations from the cylinder and the cone is less than one fifth of the smallest linear size.

In the elastic area of material deformation, compound shell structure is under the influence of variable external pressure

$$q(x) = q_0 \cdot f(x), \quad (1)$$

where  $f(x)$  – is smooth function, and axial forces  $T$  (compressing or tensile).

At the same time, it is assumed to the prevailing effect of external pressure in relation to axial compression, which leads to such buckling forms that correspond to the formation of one half-wave in the longitudinal direction and  $n$  waves in the circumferential and  $n^2 \gg 1$  [15].

Through  $\bar{s}$  and  $s$  referred to the coordinates along the forming cylindrical and conical surfaces, respectively,  $y$  – the arc coordinates for the cylinder,  $\varphi$  – the angular coordinate along the cone parallel. The middle surface of each section

of the shell design is the surface of rotation with the following functions of the radius of the parallel circle in the section, perpendicular to the axis of rotation [5; 6]:

1) for a “generalized barrel” compartment:

$$r = R \left( 1 + C_{bar} \sin \frac{\pi \bar{s}}{L} \right), \quad (2)$$

where  $L$  – the distance between the bases  $R$  – the radius of the base of the section,  $C_{bar}$  – is the relative deviation of the shell forming from the cylinder;

2) for “generalized ogive” section:

$$r = \cos \alpha \left[ s + C_{og} l_1 \sin \frac{\pi (s - l_0)}{l_1 - l_0} \right], \quad (3)$$

where  $l_0$  and  $l_1$  – the distances along the axis to the smaller and larger base,  $\alpha$  – is the angle of the taper,  $C_{og}$  – the relative deviation of the shell forming from the cone.

Restrictions on the parameters of the shells and, based on equations (2) and (3), the approximate values of the radii of the main curvature were obtained [5]:

3) for a “generalized barrel” compartment:

$$\begin{aligned} \tilde{R}_1 &= -\frac{(1 + (r')^2)^{3/2}}{r''} \approx \frac{L^2}{RC_{bar}\pi^2 \sin \frac{\pi \bar{s}}{L}}, \\ \tilde{R}_2 &= r\sqrt{1 + (r')^2} \approx R \left( 1 + C_{bar} \sin \frac{\pi \bar{s}}{L} \right), \end{aligned} \quad (4)$$

4) for a “generalized ogive” compartment:

$$\tilde{R}_1 \approx \frac{(l_1 - l_0)^2}{C_{og} l_1 \pi^2 \cos \alpha \sin \alpha \sin \Omega}, \quad \tilde{R}_2 \approx \text{ctg } \alpha (s + C_{og} l_1 \sin \Omega), \quad (5)$$

signs of  $C_{bar}$  and  $C_{og}$  determine the sign of Gaussian curvature of the middle surface  $\kappa = 1/(R_1 R_2)$  of the corresponding design compartment.

Allowing differential equations of the main stress-strain state relative to the deflection functions for each composite design compartment obtained in [5]:

$$a_4(\bar{x}) W_{bar}^{IV}(\bar{x}) + a_3(\bar{x}) W_{bar}'''(\bar{x}) + a_2(\bar{x}) W_{bar}''(\bar{x}) + a_1(\bar{x}) W_{bar}'(\bar{x}) + a_0(\bar{x}) W_{bar}(\bar{x}) = 0, \quad (6)$$

$$b_4(x) W_{og}^{IV}(x) + b_3(x) W_{og}'''(x) + b_2(x) W_{og}''(x) + b_1(x) W_{og}'(x) + b_0(x) W_{og}(x) = 0 \quad (7)$$

where  $\bar{x} = \bar{s}/L$ ,  $x = s/l_1$  and variable the coefficients  $a_i(\bar{x})$ ,  $i = \overline{1, 4}$ ,  $b_j(x)$ ,  $j = \overline{1, 4}$  of equations depend on the geometric characteristics of the shells and external loads. Equations (6) and (7) are suitable for study on the stability of the shell design under the action of variable pressure, taking into account the axial loading, for which the substitution is necessary in equation (6) and (7) of functions  $q(x)$  in the form (1). The features for the finite difference method to the equations (6), (7) solution and the initial parameter method in matrix form for taking into account the discreteness of the intermediate circular ribs, in particular, the docking one, are formulated as well.

At the same time there are conditions:

$$W_{og}(1) = W_{bar}(0), \quad W'_{og}(1) = W'_{bar}(0), \quad (8)$$

$$W''_{og}(1) + G_2^* W'_{og}(1) = W''_{bar}(0), \quad W'''_{og}(1) - G_1^* W_{og}(1) = W'''_{bar}(0), \quad (9)$$

where  $G_1^*$  and  $G_2^*$  – the dimensionless parameters of the rigidity of the splits, respectively, in the plane and from the initial curvature plane.

The condition of conjugation of the generalized “ogive” and “barrel” -shaped sections are determined by the equality of tilt tangent the angles  $\beta$  to the middle meridians of the sections, so the location of the docking splint can be considered locally conical. In this regard, the rigidity of the rib are determined by formulas:

$$G_1^* = G_1 \frac{1}{\cos^3 \beta}, \quad G_2^* = G_2 \frac{1}{\cos \beta}; \quad (10)$$

$$G_1 = \frac{n^4 (n^2 - 1)^2 (EJ)_x^{ring}}{Eh R^3}, \quad G_2 = \frac{n^2 (n^2 - 1)^2 (EJ)_z^{ring}}{EhR^3 (n^2 + 1)}. \quad (11)$$

In the study of the construction of the cylinder cone, we will follow the idea to paper [4], where the docking rib was divided into two parts corresponding to its cylindrical and conical component, as a result of which rigidity was taken into account in the form

$$\begin{aligned} G_{cyl,1} &= \frac{G_1}{2}, \quad G_{cyl,2} = \frac{G_2}{2}; \quad G_{cone,1} = \frac{G_1}{2} \frac{1}{\cos^3 \alpha}, \\ G_{cone,2} &= \frac{G_1}{2} \frac{1}{\cos \alpha}. \end{aligned} \quad (12)$$

At the same time, the conditions of the conjugation (8) through the spline were preserved, and the conditions (9) were presented in the form

$$\begin{aligned} W''_{og}(1) + G_{cone,2} W'_{og}(1) &= W''_{bar}(0) + G_{cyl,2} W'_{bar}(0), \\ W'''_{og}(1) - G_{cone,1} W_{og}(1) &= W'''_{bar}(0) + G_{cyl,1} W_{bar}(0), \end{aligned} \quad (13)$$

In the case of the design ‘cylinder-cone’ we obtain

$$W_{bar} = W_{cyl}, \quad W_{og} = W_{cone}. \quad (14)$$

**2. Numerical results analysis for the compound structure.** For the estimated implementation, the compound shell design with the characteristics is selected:  $h = 0,3$  cm,  $E = 7 \cdot 10^5$  kg/cm<sup>2</sup>,  $\nu = 0,32$ . The values  $l_1 = 182$  cm, and  $L = 2,5R$  for “generalized ogive” and “generalized barrel” shaped compartments respectively are accepted. Calculations were carried out for the case of boundary conditions corresponding to the hinge support of the ends of the compound structure.

The dimensionless load parameters (axial loading and external pressure) are introduced:

$$T^* = \frac{T}{Eh^2}, \quad q^* = \frac{q_0}{q_{cyl}}. \quad (15)$$

where  $q_{cyl}$  is the classic value of critical pressure for the cylindrical shell [8]:

$$q_{classic, cyl} = 0,92 E \left( \frac{h}{R} \right)^{5/2} \frac{R}{L}. \quad (16)$$

**2.1. The calculated parameters of the compound design for variable distribution of external pressure.** Calculations were carried out for the parameters of the shell  $l_0 = 0,45 l_1$ ,  $\alpha = 75^\circ$ , the rigidity parameters of the docking splint are  $G_1 = 5000$ ,  $G_2 = 10$ .

At the same time, two types of variable external pressure distribution are considered: according to the linear law along the axial coordinate and nonlinear one. We introduce an auxiliary function specifying external pressure having a distribution of the same intensity:

$$f(x) = \frac{\left(2 - \frac{l_0}{l_1}\right) f_1(x)}{\int_{l_0/l_1}^2 f_1(x) dx}, \quad x \in \left[\frac{l_0}{l_1}, 2\right]. \quad (17)$$

For a linear law, functions  $f_1(x) = x$  and  $f_1(x) = 2 - cx$  (here  $c$  – the parameter of the linear function) are selected.

For the nonlinear function it is accepted formulas:

$$f_1(x) = \frac{F_1(x)}{\max_{[l_0/l_1; 2]} F_1(x)} + \frac{F_2(x)}{\max_{[l_0/l_1; 2]} F_2(x)};$$

$$F_1(x) = \frac{x}{C_1 \left( x - \left( \frac{l_0}{l_1} + \frac{l_1 - l_0}{l_1} x_1 \right) \right)^{2s} + 1},$$

$$F_2(x) = \frac{C_3 \cdot x}{C_2 \left( x - \left( \frac{l_1 + l_0}{l_1} + \frac{l_1 - l_0}{l_1} x_2 \right) \right)^{2s} + 1} \quad (18)$$

where  $C_1, C_2 > 0$  – quite large function parameters;  $C_3 \in [0, 1]$ ;  $s = 1, 2, \dots$

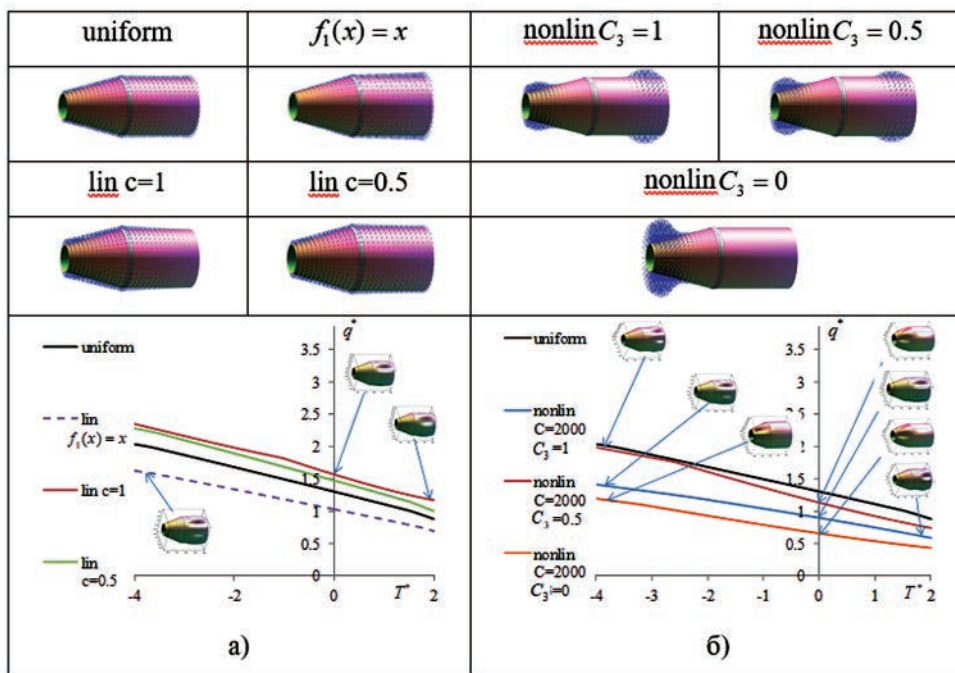
The auxiliary function  $f(x)$  in calculations is used to determine the calculated functions on the generalized ‘barrel’ and ‘ogive’ compartments in their own dimensionless coordinates:

$$f_{bar}(\bar{x}) = f(\bar{x} + 1), \quad \bar{x} \in [0; 1]; \quad f_{og}(x) = f(x), \quad x \in \left[\frac{l_0}{l_1}; 1\right]. \quad (19)$$

**2.2. The effect of variable distribution of external pressure on the ‘cylinder-cone’ design stability under combine loading.** The dependences  $q^*$  on  $T^*$  for the ‘cylinder-cone’ structure on parameters  $l_0 = 0,45 l_1$ ,  $\alpha = 75^\circ$ , and  $G_1 = 5000$ ,  $G_2 = 10$  (docking rib rigidity) under the action of axial loading and variable distributed external pressure along every compartment with visualization of the buckling modes are shown on fig. 1 (1(a)-linear and 1(b)-nonlinear laws).

It is necessary to note that in contrast to the results obtained for the conical three-layer shell in paper [14], where a minor effect was found on the stability of the linear distribution of external pressure  $f_1(x) = 2 - cx$ , for the investigated compound designs, the influence of the linear law was significant and leads to increasing and to a decrease in critical pressure depending on the structure parameters.

Dependencies in fig. 1 for the “cylinder-cone” design demonstrate the deviation of the values of the critical pressure to 20% with respect to the case of uniform pressure (in the considered range of loads and the parameters of the shells). In this case, the external pressure distributed in accordance with the functions  $f_1(x) = x$  and  $f_1(x) = 2 - cx$  leads to decreasing and increasing critical pressure respectively.



**Fig. 1. The dependences  $q^*$  on  $T^*$  for the compound ‘cylinder-cone’ structure**

Analysis of critical loads and visualization of the nature of wave-formation of the studied compound design showed that the linear distribution of external pressure changes only into the number of waves in the circumferential direction, ranging from  $n=5$  to  $n=8$ , depending on the external pressure distribution functions and axial loading, in corresponding to formulated approach investigation.

The nonlinear law of the distribution of external pressure on the ‘cylinder-cone’ design gives more significant deviations both in the values of buckling pressures (up to 42%) and in post critical wave formation surface.

It is necessary to note, that in fig. 1 b) calculated parameters of the function of non-linear external pressure with two in the same height of bursts on the ‘‘cylinder’’ and ‘‘cone’’ (which corresponds to  $C_3 = 1$ ) give a decrease in critical pressure to 31% compared with evenly distributed.

In the case of a nonlinear distribution of pressure with amplitude on the ‘‘cone’’ two times greater than on the ‘‘cylinder’’ (here  $C_3 = 0.5$ ), a relatively small deviation is observed in the value of the critical pressure (up to 15%) with respect to uniformly distributed external pressure. In the range of changes in the axial force  $T^*$  from  $-2,7$  to  $2$ , the crest of the buckling wave is placed on the conical compartment, while  $n = 5$ . At  $T^* < -2,7$  the wave number increases to  $n = 9$  and the crest of the wave shifts to the cylindrical compartment.

2.3. The influence of the linear laws distribution of external pressure on the buckling structure with nonzero curvature compartments. The dependences  $q^*(T^*)$  for

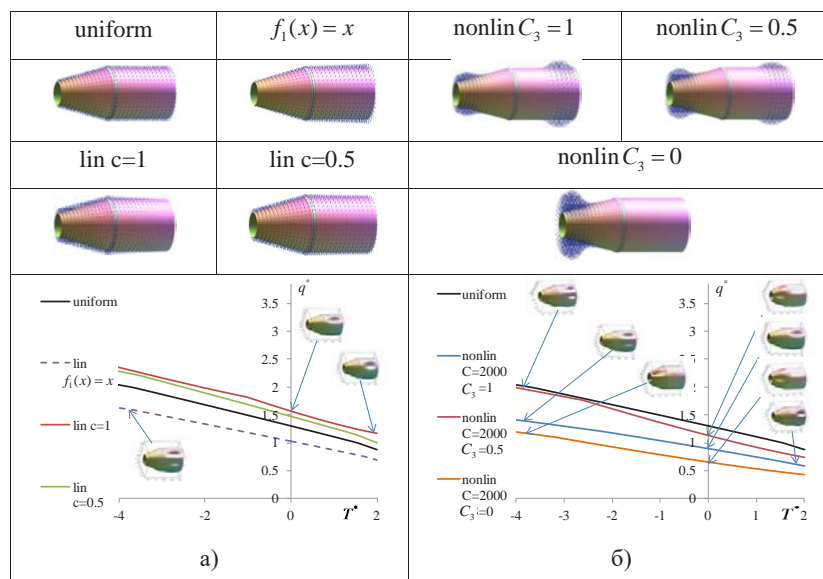
compartments with various combinations of curvature signs of middle meridians are shown in fig. 2:

- 1) in Fig. 2 (a) considered the design ‘barrel-pseudo-ogive’ for
- 2)  $C_{bar} = 0.25, C_{og} = -0.03019235707$ ;
- 3) in Fig. 2 (b) – ‘barrel – ogive’ for
- a.  $C_{bar} = 0.137, C_{og} = 0.06258642604$ ;
- 4) in Fig. 2 (c) – ‘pseudo-barrel-ogive’ for
- 5)  $C_{bar} = -0.07, C_{og} = 0.2325440198$ .

Analyzing the values of critical pressures with a linear distribution of external pressure (fig. 2), it can be concluded that in the case of a compressive force, the convex shell leads more efficiently. When tensile designs, the structure with negative values of Gaussian curvature on one of the compartments is more efficient than the ‘barrel-ogive’ shell structure.

The designs with a negative curvature on one of the compartments demonstrate sharp changes in behavior, which is characterized by a jump-shaking absolute increase in the wave number  $n$  (flow of dependence curve  $q^*(T^*)$ ) and with a sharp change in the form of wave formation. Although the considered design ‘barrel – ogive’ mainly has predictable behavior when tensile-compression with a gradually changing wave number  $n$  in redistribution from 9 to 12, however, at  $c = 1$ , a similar situation with buckling modes appears as in the case of the presence of negative curvature on one of the compartments.

Buckling modes for a convex structure selected from the condition of equal resistance at uniform pressure and the corresponding axial force (fig. 2 (b)) are considered. The most striking change in the post-buckling form of a shell during compression gives a shift of the maximum value of a linearly distributed external pressure toward the ‘‘generalized barrel’’ ( $f_1(x) = x$ ) compartment. In this



**Fig. 2. Dependences  $q^*(T^*)$  for compartments with various combinations of curvature signs of middle meridians**

case the crest of the buckling form replace to the end of the “barrel” compartment. Stretching results in a more uniformly distributed waveform located along the barrel-shaped section. In this case, the wave transition to the conical part does not occur for this design.

The linear distribution of external pressure, built on the basis of a function  $f_1(x) = 2 - cx$ , for a parameter  $c = 0.5$  gives similar patterns in wave formation, as with uniform external pressure, but with a shift of similar forms towards stretching. For  $c = 1$ : in the case of the axial force  $T^* > -1$ , the crest of the waves is distributed near the rib with a greater or less degree of localization near the rib, already when  $T^* < -2$  the comb is placed on the “barrel” with the capture of the rib and the “ogive” small part. For the  $c = 0.5$  wave formation parameter with the arrangement of the crest of the wave of the release on the “ogive” compartment was not detected.

If  $c = 1$  and the axial load parameter  $T^* > -1$ , the crest of the wave is visualized in the spangling area. If parameter  $T^* \in (-3.3; -1.2)$ , the comb is shifted toward the “ogive” compartment. In these cases, the wave number  $n$  is equal to 9 or 10. A further increasing of the axial compression leads to a sharp changing in the wave number to a value  $n = 12$  and the crest of the wave moves to the “barrel-shaped” compartment.

2.4 The influence of external pressure nonlinear distribution for the compound design with a nonzero middle surface compartment curvature. The effect of the parameters  $C_1, C_2$  distribution function of the nonlinear law of external pressure with equal values of the maximum pressure ( $C_3 = 1$ ) on the value of the critical pressure for “generalized barrel-ogive” structure was studied.

Calculations for parameters  $l_0 = 0,45 l_1, \alpha = 75^\circ, G_1 = 5000, G_2 = 10$ . For the first step choose  $x_1 = 0,05; x_2 = 0,6; s = 2, C_2 = C_1 = C$ . The dependences  $q^*$  on  $T^*$  for the shells of various shape combinations of the “generalized barrel-ogive structure” are shown in fig. 3:

– in Fig. 3 (a) considered the ‘barrel-pseudo-ogive’ design for

$$C_{bar} = 0.25, C_{og} = -0.03019235707;$$

– in Fig. 3 (b) – ‘barrel – ogive’ design for

$$C_{bar} = 0.137, C_{og} = 0.06258642604;$$

– in fig. 3 (c) – ‘pseudo-barrel-ogive’ design for

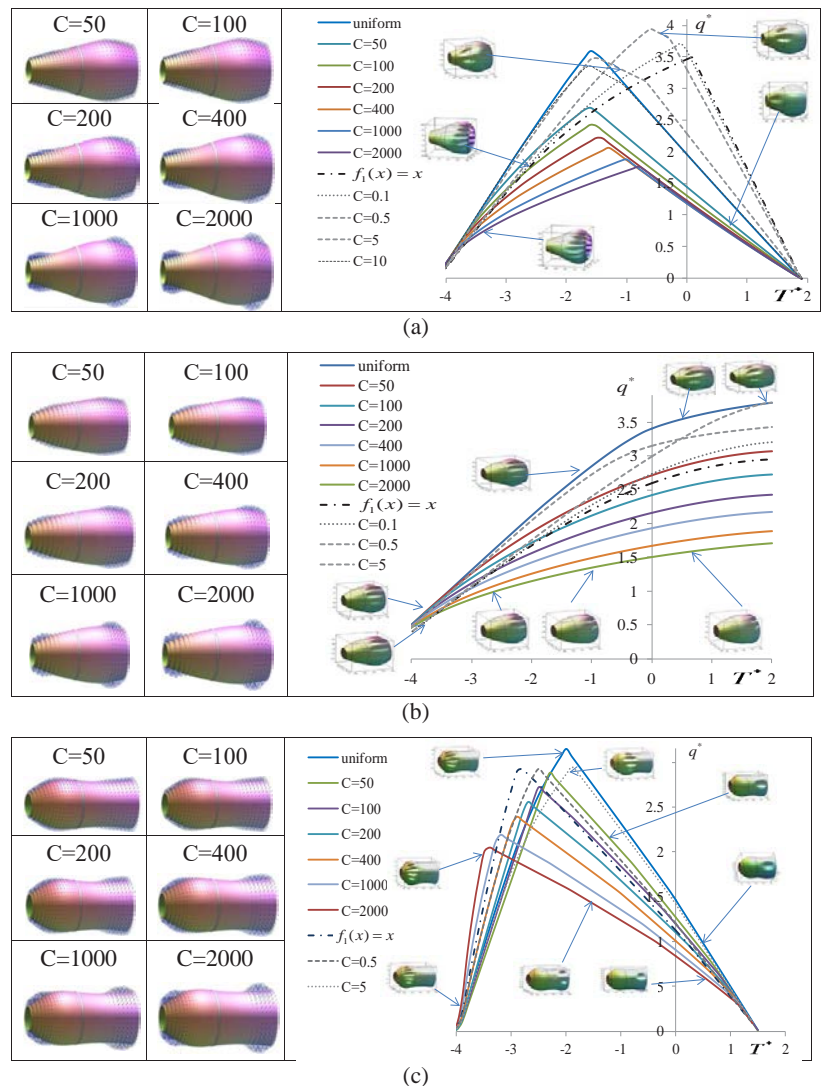
$$C_{bar} = -0.07, C_{og} = 0.2325440198.$$

For the nonlinear distribution parameters of external pressure  $x_1 = 0,1; x_2 = 0,9; s = 1, C_2 = C_1 = C$  the parameters of the shell  $l_0 = 0,45 l_1, \alpha = 75^\circ, C_{bar} = 0,25, C_{og} = -0.03019235707$  (“barrel-pseudo-ogive”) and the rigidity of the docking rib  $G_1 = 5000, G_2 = 10$ . The dependence  $q^*$  on  $T^*$  for the various values of the parameter  $C$  of given nonlinear law shown in fig. 4.

A distinctive feature of the external pressure distribution by laws corresponding to fig. 4 with parameters  $C = 1000, C = 2000$  is its approach to the effects of localized external pressure.

**Conclusions**

1. The buckling problem for the compound shell structure with deviation compartments from the cylinder (“generalized barrel”) and the cone (“generalized ogive”), in particularly, with



**Fig. 3. The dependences  $q^*$  on  $T^*$  for the compartments of various shape combinations of the “generalized barrel-ogive” structure**

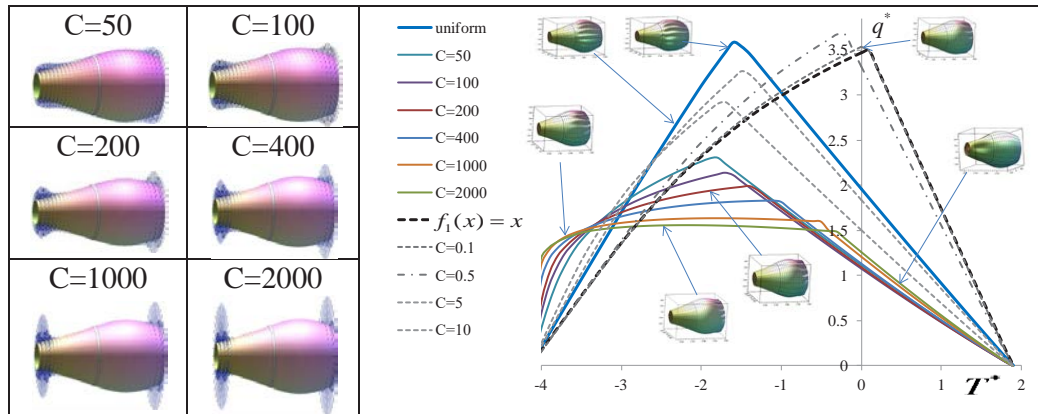


Fig. 4. Critical loads dependences for the different parameters C

different signs of curvature of the middle meridians is discussed. An analysis of the influence of Gaussian curvature of the middle surface of the design for stability under combined loading was carried out with implementation of buckling modes visualization for various parameters of the structure geometry and the functions of the external pressure distribution.

2. Two type laws (linear and non-linear) identical intensity of variable distribution of external pressure are investigated. It is shown that in the case of axial compressing is more effective is a “convex” compound

structure, and in tension –“convex-concave” structure. At exposed to the design of non-linear external pressure distribution with one maximum on the ‘ogive’ more efficiently from the point of view stability is the ‘barrel-pseudo-ogive’ design.

3. A nonlinear distribution of external pressure in some cases leads to situation when the wave of the bulking has a significant displacement to one of the ends. The smoothing of the buckling wave, necessary from a technological point of view, when designing a real structure, possibly by changing the curvature parameters of the middle surface meridian.

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