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SOLUTION OF THE FIRST BASIC BOUNDARY VALUE PROBLEM OF THE PLANE ELASTICITY THEORY FOR A MULTILAYER BASE WITH ORTHOTROPIC LAYERS

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The paper deals with the problem of determining stresses and displacements at the points of a multilayer base consisting of orthotropic layers and coupled to a half-space. The external loads on the top layer are known, such that the deformation of the body becomes flat. At infinity, the stresses are zero.

This paper presents a brief review of scientific studies that highlight methods and approaches to solving problems related to the theory of elasticity for studying the stress-strain state of multilayer bodies, plates, plates, and strips.

The article formulates an algorithm for analytically solving the problem for a multilayer base, in which all the basic equations of the problem and boundary conditions are subjected to a direct Fourier transform. The stress function is found as a solution of the analog of a biharmonic differential equation in the space of transformants in the case of an orthotropic material.

The relationships between the stress function transformant and the stress and displacement transformants are established. For each layer, four auxiliary functions are introduced that are associated with the stress and displacement transformants of points on the surface of the layers. From the conditions on the common boundaries between the layers, recurrent relations are constructed that express the auxiliary functions of the lower layer through the functions of the previous layer. By expressing the four auxiliary functions for the first layer, we can find similar functions for any layer using recurrent formulas.

After substituting the found expressions into the stress and displacement transforms and applying the inverse Fourier integral transform, we obtain the true values of stresses and displacements at the points of the multilayer orthotropic base.

The proposed algorithm takes into account the peculiarities of the properties of the orthotropic material and allows us to obtain analytical solutions of the stress-strain state in each layer of the base.

РОЗВ'ЯЗОК ПЕРШОЇ ОСНОВНОЇ КРАЙОВОЇ ЗАДАЧІ ПЛОСКОЇ ТЕОРІЇ ПРУЖНОСТІ ДЛЯ БАГАТОШАРОВОЇ ОСНОВИ З ОРТОТРОПНИМИ ШАРАМИ

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Ключові слова: *напружено-деформований стан, багатошарова основа, ортотропний шар, плоска деформація, інтегральне перетворення Фур'є.*

У статті розглядається задача про визначення напружень і переміщень в точках багатошарової основи, що складається з ортотропних шарів яка зчеплена з півпростором. Відомі зовнішні навантаження на верхньому шарі, такі що деформація тіла стає плоскою. На нескінченності напруження дорівнюють нулю.
В роботі наведено короткий огляд наукових досліджень, які висвітлюють методи та підходи до вирішення завдань, пов'язаних з теорією пружності для дослідження напружено-деформованого стану багатошарових тіл, плит, пластин і смуг.
В статті сформульовано алгоритм аналітичного розв'язання поставленої задачі для багатошарової основи, в якому всі основні рівняння задачі та граничні умови піддаються прямому перетворенню Фур'є. Функція напружень знаходиться як розв'язок аналогу бігармонічного диференціального рівняння в просторі трансформант на випадок ортотропного матеріалу.
Встановлюються взаємозв'язки між трансформантою функції напружень та трансформантами напружень і переміщень. Для кожного шару введено чотири допоміжні функції, які пов'язані з трансформантами напружень і переміщень точок на поверхні шарів. З умов на спільних межах між шарами побудовано рекурентні співвідношення, що виражають допоміжні функції нижнього шару через функції попереднього шару. Виражаючи четвірку допоміжних функцій для першого шару, можемо знайти аналогічні функції для довільного шару за рекурентними формулами.
Після підстановки знайдених виразів в трансформанти напружень та переміщень і застосування оберненого інтегрального перетворення Фур'є ми отримуємо істинні значення напружень і переміщень в точках багатошарової ортотропної основи.
Запропонований алгоритм враховує особливості властивостей ортотропного матеріалу і дозволяє отримувати аналітичні рішення напружено-деформованого стану в кожному шарі основи.

Introduction. The problem of determining the stress-strain state in complex multilayer systems is relevant and important for many fields of science and technology. For example, in industrial and civil engineering, similar problems arise in the calculation of structures, road pavements, and foundations when structures are considered as multilayer bodies or arrays, multilayer plates and bases, in particular, those lying on an elastic or rigid half-space.

To date, many different methods have been developed for the calculation of layered structures. For example, in [1], a new approach (the macroelement method) was developed to calculate the stress-strain state of orthotropic slabs resting on an elastic Winkler base. The authors showed that this approach provides a more accurate solution compared to the finite element method.

The modeling of an elastic base by the Pasternak equation, which gives a more realistic view of the deformation of the base, is discussed in [2]. It is devoted to the development of an analytical and numerical method for solving the problem of modeling the stress-strain state of layered orthotropic plates on an elastic Pasternak base.

An assessment of mixed and classical theories on the global and local responses of multilayer orthotropic plates is given in [3]. In this paper, the authors conclude that the application of Reisner's mixed variational theorem to mixed problems of elasticity theory gives an advantage in the accuracy of calculations over the classical principle of possible displacements.

Paper [4] provides a general overview of the theories, as well as an analysis of the accuracy and efficiency of various theories for studying the deformed state of laminated plates and the corresponding finite element models. The authors have shown that global-local theories are more effective in predicting transverse shear stresses compared to other theories (zigzag theory, Reddy theory).

Paper [5] presents a solution to the problem of axisymmetric torsion of a multilayer plate with elastic connections between the layers using the method of compliance functions and the Hankel integral transform.

The application of the Fourier integral transform and the method of compliance functions is described in [6; 7]. In [6], this method was applied to solve the problem of plane deformation of an isotropic multilayer plate with elastic connections between the layers, and in [7], to determine the contact zone and contact stresses between an isotropic strip and an elastic half-plane.

The solution of the basic boundary value problems of the plane theory of elasticity for a transversally isotropic multilayer base by the method of the integral Fourier transform with the construction of compliance functions is given in [8].

This literature review confirms the relevance of modeling and analyzing the stress-strain state of layered structures. It demonstrates the effectiveness of the method of compliance functions in solving problems of elasticity theory with isotropic and transversally isotropic layers, but it has not been applied to structures made of orthotropic materials.

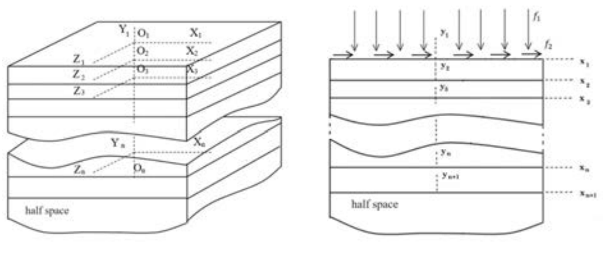
Therefore, the purpose of this paper is to extend the method of compliance functions using the integral Fourier transform to solve the first basic boundary value problem of plane elasticity for a multilayer base with orthotropic layers.

Statement of the problem. Let's consider a package of *n* layers that is connected to a half-space. We call this structure a multilayer base. Each layer is assumed to be homogeneous, weightless, orthotropic, and characterized by a thickness *h* and elastic constants – Poisson's ratios *ν* and Young's modulus's *E*. The deformation of the multilayer base is plane. It is necessary to determine the stresses and displacements in the layers of the base if the loads on the surface are known.

The layers in the base are numbered from top to bottom, starting with one, with the layer that lies on the half-space having the number n, and the half-space being numbered $n+1$. The layer with the number 1 will be called the top layer, and the layer with the number n will be called the bottom layer (Fig. 1a). All values related to the layers of the bases will be denoted by the upper or lower index $k=\overline{1,n}$ (usually lower). For example, the thickness of the layer with the number k is denoted by h_k.

The materials of the layers and the half-space are characterized by elastic constants – Poisson's ratios ν_{ij}^k and Young's modulus's E_i^k, where де $i,j=1,2$, $k=\overline{1,n+1}$.

For each layer and half-space, we introduce coordinated local rectangular Cartesian coordinate systems $O_kX_kY_kZ_k$ (k is the layer number), as shown in Figure 1a. All the origin of the local coordinate systems are located on a single line perpendicular to the surface of the bases. The directions of all O_kX_k axes are parallel to each other, as well as the directions of the O_kZ_k axes. The local coordinate planes $O_kX_kZ_k$ coincide with the upper planes of the corresponding layer. For half-space, the local system is introduced in the same way.



(a) (b)

Fig. 1. Multilayer orthotropic base

The external load $f_1(x_1), f_2(x_1)$ is such that the deformation of the base and half-space is flat, so we proceed to a two-dimensional formulation of the problem (Fig. 1b). We will assume that the movements of the body points occur parallel to the $O_1X_1Y_1$ plane.

Geometric areas that occupy layers and half-space:

$$G_k(x_k, y_k): \{-\infty < x_k < +\infty, -h_k \le y_k \le 0\},$$

$$G_{n+1}(x_{n+1}, y_{n+1}): \{-\infty < x_{n+1} < +\infty, -\infty < y_{n+1} \le 0\}.$$

For a layer with number k, the upper bound $y_k = 0$ is described by $-\infty < x_k < +\infty$, the lower bound $y_k = -h_k$, $-\infty < x_k < +\infty$.

Boundary conditions:

1) boundary $y_1 = 0$:

$$\sigma_y^1(x_1, 0) = f_1(x_1), \tau_{xy}^1(x_1, 0) = f_2(x_1); \quad (1)$$

2) common boundaries between the layers $k = \overline{1, n-1}$:

$$\sigma_y^k(x_k, -h_k) = \sigma_y^{k+1}(x_{k+1}, 0), \tau_{xy}^k(x_k, -h_k) = \tau_{xy}^{k+1}(x_{k+1}, 0),$$

$$u_x^k(x_k, -h_k) = u_x^{k+1}(x_{k+1}, 0), u_y^k(x_k, -h_k) = u_y^{k+1}(x_{n+1}, 0); (2)$$

3) common boundary of the lower layer and the absolutely rigid half-plane:

$$u_x^n(x_n, -h_n) = u_x^{n+1}(x_{n+1}, 0) = 0, u_y^n(x_n, -h_n) = u_y^{n+1}(x_{n+1}, 0) = 0; (3)$$

4) at infinity, $k = \overline{1, n+1}$:

$$\lim_{x_k^2 + y_k^2 \to \infty} \sigma_x^k(x_k, y_k) = 0, \lim_{x_k^2 + y_k^2 \to \infty} \sigma_y^k(x_k, y_k) = 0, \lim_{x_k^2 + y_k^2 \to \infty} \tau_{xy}^k(x_k, y_k) = 0. (4)$$

Methods. To determine the stress-strain state of bodies, we will apply the method of one-dimensional integral Fourier transform to the obtained stress function $\varphi(x, y)$ (the algorithm is described in [9]). This method, in combination with the method of compliance functions for isotropic materials, was proposed and developed in [10; 11]. The extension of this method to the case of an orthotropic half-plane is discussed in [12].

To determine the stress-strain state of bodies, we will apply the method of the one-dimensional integral Fourier transform [13] to the stress function $\varphi(x, y)$ with variable x and transformation parameter ξ:

$$\bar{\varphi}(\xi, y) = \int_{-\infty}^{\infty} \varphi(x, y) \cdot e^{i\xi x} dx, \quad \varphi(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\varphi}(\xi, y) \cdot e^{-i\xi x} d\xi. \quad (5)$$

The first formula defines the direct one-dimensional integral Fourier transform for the function $\varphi(x, y)$, and the second defines the inverse. The function $\varphi(\xi, y)$ is called the Fourier transform of the function $\varphi(x, y)$, for which the property [10] is true:

$$\int_{-\infty}^{\infty} \frac{\partial^k \varphi(x, y)}{\partial x^k} \cdot e^{i\xi x} dx = (-i\xi)^k \cdot \bar{\varphi}(\xi, y). \quad (6)$$

The solution to the boundary value problem is sought in the space of transformants of the one-dimensional integral transform. In this case, all the basic equations of the problem and boundary conditions are directly transformed by the one-dimensional Fourier integral transform.

Find the solution of the analog of the biharmonic differential equation of a plane problem for an orthotropic material [14], to which we apply the Fourier integral transform:

$$A_1 \cdot \frac{d^4 \bar{\varphi}}{dy^4} - 2A_3 \xi^2 \cdot \frac{d^2 \bar{\varphi}}{dy^2} + A_2 \xi^4 \cdot \bar{\varphi} = 0,$$

where

$$A_1 = c_{11}, A_2 = c_{22}, A_3 = \frac{c_{33} - c_{12}}{2}, c_{11} = = \frac{1 - \nu_{xz} \cdot \nu_{zx}}{E_x}, c_{22} = \frac{1 - \nu_{yz} \cdot \nu_{zy}}{E_y}, c_{33} = \frac{1}{G_{xy}},$$

$$c_{12} = c_{21} = \frac{\nu_{xy} + \nu_{xz} \cdot \nu_{zy}}{E_y} = \frac{\nu_{yx} + \nu_{zx} \cdot \nu_{yz}}{E_x}, \ G_{xy} = \frac{\sqrt{E_x \cdot E_y}}{2 \cdot (1 + \sqrt{\nu_{xy} \cdot \nu_{yx}})} -$$

elasticity constants in Hooke's law, $\bar{\varphi} = \bar{\varphi}(\xi, y)$ – Fourier transform on the variable x from $\varphi(x, y)$

Let's write the transformant of the stress function $\overline{\varphi_{2k}}(\xi, y), k = \overline{1, n+1}$.

$$\overline{\varphi_{2k}}(\xi, y) = A_{2k}(\xi)\, sh(ry\sqrt{a_k}) + B_{2k}(\xi)\sqrt{a_k} y\, sh(ry\sqrt{a_k}) + C_{2k}(\xi)\, ch(ry\sqrt{a_k}) + D_{2k}(\xi)\sqrt{a_k} y\, ch(ry\sqrt{a_k}), \quad (7)$$

where $r = |\xi|, \sqrt{a_k} = \sqrt{\frac{A_3}{A_1}}, k = \overline{1, n+1}.$

The stress function $\varphi_{2k}(x, y)$ is chosen to satisfy the conditions exactly:

$$\sigma_x^k(x, y) = \frac{\partial^2 \varphi_{2k}}{\partial y^2}, \quad \sigma_y^k(x, y) = \frac{\partial^2 \varphi_{2k}}{\partial x^2}, \quad \tau_{xy}^k(x, y) = -\frac{\partial^2 \varphi_{2k}}{\partial x \partial y}. \quad (8)$$

Applying property (6) to formulas (8), we obtain:

$$\overline{\sigma_x^k}(\xi, y) = \frac{\partial^2 \overline{\varphi_{2k}}}{\partial y^2}, \quad \overline{\sigma_y^k}(\xi, y) = -\xi^2 \overline{\varphi_{2k}}, \quad \overline{\tau_{xy}^k}(\xi, y) = i\xi \cdot \frac{\partial \overline{\varphi_{2k}}}{\partial y}. \quad (9)$$

Applying the Fourier transform property to the formulas $\gamma_{xy} = c_{33}\tau_{xy}$, $\varepsilon_x = c_{11}\sigma_x - c_{12}\sigma_y$ we obtain the displacement transformants $\overline{u_x^k}(\xi, y)$, $\overline{u_y^k}(\xi, y)$:

$$\overline{u_x^k} = \frac{i}{\xi}\left(c_{11}\overline{\sigma_x^k}(\xi, y) - c_{12}\overline{\sigma_y^k}(\xi, y)\right), \quad \overline{u_y^k} = \frac{i}{\xi}\left(c_{33}\overline{\tau_{xy}^k}(\xi, y) - \frac{d\overline{u_x^k}}{dy}\right). \quad (10)$$

The boundary conditions in the transformant space take the form:

1) boundary $y_1 = 0$:

$$\overline{\sigma_y^1}(\xi, 0) = \overline{f_1}(\xi), \quad \overline{\tau_{xy}^1}(\xi, 0) = \overline{f_2}(\xi); \quad (1')$$

2) common boundaries between the layers $k = \overline{1, n-1}$:

$$\overline{\sigma_y^k}(\xi, -h_k) = \overline{\sigma_y^{k+1}}(\xi, 0), \quad \overline{\tau_{xy}^k}(\xi, -h_k) = \overline{\tau_{xy}^{k+1}}(\xi, 0),$$
$$\overline{u_x^k}(\xi, -h_k) = \overline{u_x^{k+1}}(\xi, 0), \quad \overline{u_y^k}(\xi, -h_k) = \overline{u_y^{k+1}}(\xi, 0); \quad (2')$$

3) common boundary of the lower layer and the absolutely rigid half-plane:

$$\overline{\sigma_y^n}(\xi, -h_n) = \overline{\sigma_y^{n+1}}(\xi, 0), \quad \overline{\tau_{xy}^n}(\xi, -h_n) = \overline{\tau_{xy}^{n+1}}(\xi, 0) = 0,$$
$$\overline{u_x^n}(\xi, -h_n) = \overline{u_x^{n+1}}(\xi, 0) = 0, \quad \overline{u_y^n}(\xi, -h_n) = \overline{u_y^{n+1}}(\xi, 0) = 0; \quad (3')$$

4) at infinity, $k = \overline{1, n+1}$:

$$\lim_{x_k^2 + y_k^2 \to \infty} \overline{\sigma_x^k}(\xi, y_k) = 0, \lim_{x_k^2 + y_k^2 \to \infty} \overline{\sigma_y^k}(\xi, y_k) = 0, \lim_{x_k^2 + y_k^2 \to \infty} \overline{\tau_{xy}^k}(\xi, y_k) = 0. \quad (4')$$

Substituting (7) into the boundary condition formulas (2'), we obtain:

$$\overline{\sigma_y^k}(\xi, -h_k) = \xi^2 ch(rh_k\sqrt{a_k}) \cdot (D_{2k}\sqrt{a_k} h_k - C_{2k}) - \xi^2 sh(rh_k\sqrt{a_k}) \cdot (B_{2k}\sqrt{a_k} h_k - A_{2k}),$$

$$\overline{\sigma_y^{k+1}}(\xi, 0) = -\xi^2 C_{2k}, \quad \overline{\tau_{xy}^{k+1}}(\xi, 0) = i\xi\sqrt{a_k}(rA_{2k} + D_{2k}),$$

$$\overline{\tau_{xy}^k}(\xi, -h_k) = i\xi ch(rh_k\sqrt{a_k}) \cdot \left((rA_{2k} + D_{2k})\sqrt{a_k} - rB_{2k} a_k h_k\right)$$

$$- i\xi sh\left(rh_k\sqrt{a_k}\right)\cdot\left((rC_{2k}+B_{2k})\sqrt{a_k}-rD_{2k}a_kh_k\right),$$

$$\overline{u_x^k}(\xi,-h_k)=\left(r^2C_{2k}c_{k12}+ra_kc_{k11}(rC_{2k}+2B_{2k})-r^2D_{2k}a_k\sqrt{a_k}h_kc_{k11}\right.$$
$$\left.-r^2D_{2k}\sqrt{a_k}h_kc_{k12}\right)\cdot\frac{ich\left(rh_k\sqrt{a_k}\right)}{\xi}+\frac{ish\left(rh_k\sqrt{a_k}\right)}{\xi}$$
$$\cdot\left(r^2B_{2k}\sqrt{a_k}h_kc_{k12}+r^2B_{2k}a_k\sqrt{a_k}h_kc_{k11}-r^2A_{2k}c_{k12}\right.$$
$$\left.-ra_kc_{k11}(rA_{2k}+2D_{2k})\right),$$

$$\overline{u_x^{k+1}}(\xi,0)=\frac{i}{\xi}\cdot\left(r^2C_{2k}c_{k12}+r^2C_{2k}a_kc_{k11}+2rB_{2k}a_kc_{k11}\right),$$

$$\overline{u_y^k}(\xi,-h_k)=\left(a_k\sqrt{a_k}c_{k11}(rA_{2k}+3D_{2k})-\sqrt{a_k}(c_{k33}-c_{k12})(rA_{2k}+D_{2k})\right.$$
$$\left.-rB_{2k}a_kh_k(a_kc_{k11}+c_{k12}-c_{k33})\right)\cdot ch\left(rh_k\sqrt{a_k}\right)$$
$$+\left(\sqrt{a_k}(c_{k33}-c_{k12})(rC_{2k}+B_{2k})-a_k\sqrt{a_k}c_{k11}(rC_{2k}+3B_{2k})\right.$$
$$\left.+rD_{2k}a_kh_k(a_kc_{k11}+c_{k12}-c_{k33})\right)\cdot sh\left(rh_k\sqrt{a_k}\right),$$

As in the case of an isotropic material described in [10, 11], we introduce auxiliary functions $\alpha_k(\xi)$, $\delta_k(\xi)$, $\beta_k(\xi)$, $\gamma_k(\xi)$, $k=\overline{1,n-1}$, for each layer, which are associated with conditions on the boundary $y_k=0$, $k=\overline{1,n-1}$.

$$\alpha_k(\xi)=\overline{\sigma_y^k}\big|_{y=0},\qquad \delta_k(\xi)=-\frac{i\xi}{r\sqrt{a_k}}\cdot\overline{\tau_{xy}^k}\big|_{y=0},$$

$$\beta_k(\xi)=\frac{r}{\sqrt{a_k}}\cdot\overline{u_y^k}\big|_{y=0},\qquad \gamma_k(\xi)=-i\xi\cdot\overline{u_x^k}\big|_{y=0}.$$

Considering formulas (9)–(10), we have:

$$\delta_k(\xi)=r(rA_{2k}+D_{2k}),\beta_k(\xi)=2rD_{2k}a_kc_{k11}+(a_kc_{k11}+c_{k12}-c_{k33})\delta_k(\xi),$$
$$\alpha_k(\xi)=-\xi^2C_{2k},\qquad \gamma_k(\xi)=(a_kc_{k11}+c_{k12})\cdot\alpha_k(\xi)+2rB_{2k}a_kc_{k11}.\quad(11)\qquad(11)$$

Let us express by and substitute them into the stress and displacement transformants (9)–(10):

$$A_{2k}=\frac{\delta_k(3a_kc_{k11}+c_{k12}-c_{k33})-\beta_k}{2r^2a_kc_{k11}},\qquad B_{2k}=\frac{\gamma_k-\alpha_k(a_kc_{k11}+c_{k12})}{2ra_kc_{k11}},$$
$$C_{2k}=-\frac{\alpha_k}{r^2},\qquad D_{2k}=\frac{\beta_k-\delta_k(a_kc_{k11}+c_{k12}-c_{k33})}{2ra_kc_{k11}}.\qquad(12)$$

From the conditions on the common boundary of the lower layer n and the half-plane $n+1(3')$, we obtain recurrence relations that express all half-plane functions $\alpha_{n+1}(\xi)$, $\delta_{n+1}(\xi)$, $\beta_{n+1}(\xi)$, $\gamma_{n+1}(\xi)$ through the lower layer functions $\alpha_n(\xi)$, $\delta_n(\xi)$, $\beta_n(\xi)$, $\gamma_n(\xi)$. Let us express $\alpha_n, \delta_n, \beta_n, \gamma_n$ by α_k, δ_k, β_k, γ_k, $k<n$ using a system of two linear homogeneous equations:

$$a_{11}(p)\alpha_k(\xi)+a_{12}(p)\beta_k(\xi)+a_{13}(p)\gamma_k(\xi)+a_{14}(p)\delta_k(\xi)=0,$$

$$a_{21}(p)\alpha_k(\xi)+a_{22}(p)\beta_k(\xi)+a_{23}(p)\gamma_k(\xi)+a_{24}(p)\delta_k(\xi)=0,\quad(14)$$

where $a_{ij}(p)=a_{ij}^k(p), i=1,2, j=\overline{1,4}$ are the corresponding coefficients in $\overline{u_x^k}(\xi,-h_k)$, $\overline{u_y^k}(\xi,-h_k)$ at $\alpha_k(\xi)$, $\delta_k(\xi)$, $\beta_k(\xi)$, $\gamma_k(\xi)$.

By solving system (14) with respect to $\beta_k(\xi), \gamma_k(\xi)$, we obtain:

$$\beta_k(\xi)=A_k(p)\alpha_k+A_{\tau k}(p)\delta_k,\ \gamma_k(\xi)=B_k(p)\alpha_k+B_{\tau k}(p)\delta_k,\quad(15)$$

where $A_k(p), A_{\tau k}(p), B_k(p), B_{\tau k}(p)$ – are the compliance functions,

$$A_k(p)=\frac{a_{13}a_{21}-a_{11}a_{23}}{a_{12}a_{23}-a_{13}a_{22}},A_{\tau k}(p)=\frac{a_{13}a_{24}-a_{14}a_{23}}{a_{12}a_{23}-a_{13}a_{22}},$$

$$B_k(p)=\frac{a_{11}a_{22}-a_{12}a_{21}}{a_{12}a_{23}-a_{13}a_{22}},B_{\tau k}(p)=\frac{a_{14}a_{22}-a_{12}a_{24}}{a_{12}a_{23}-a_{13}a_{22}}.$$

From the conditions of the problem, we know the external loads on the upper layer $\overline{\sigma_y^1}(\xi,0)=\overline{f_1}(\xi)$, $\overline{\tau_{xy}^1}(\xi,0)=\overline{f_2}(\xi)$, and therefore the two functions $\alpha_1(\xi)$ and $\delta_1(\xi)$ are known. Substituting them into the formulas described above, we obtain $\beta_1(\xi), \gamma_1(\xi)$. Substitute the found four into the recurrent formulas and find α_k, δ_k, β_k, γ_k. The resulting expressions of the unknown functions are substituted into the stress and displacement transformants and then subjected to the inverse Fourier integral transform to obtain their true values.

Algorithm for solving the problem.

1) Find the transformant of the stress function (7) and express the transformants of stresses (9) and displacements (10) using the formulas.

2) Express the unknown functions $A_{2k}(\xi), B_{2k}(\xi), C_{2k}(\xi), D_{2k}(\xi)$ in the stress and displacement transformants through the four auxiliary functions $\alpha_k(\xi)$, $\delta_k(\xi)$, $\beta_k(\xi), \gamma_k(\xi)$, which are associated with the conditions at the boundary $y_k=0(12)$.

3) Find the recurrence relations (13) from the conditions on the joint boundaries between the layers (2') and express $\beta_k(\xi), \gamma_k(\xi)$ (15) from the conditions on the joint boundary of the lower layer n and the half-plane $n+1$ $(3')$.

4) Calculate the functions $\alpha_1(\xi)$, $\delta_1(\xi)$ according to the boundary conditions $(1')$ and express the functions $\beta_1(\xi), \gamma_1(\xi)$ using the formulas (14).

5) Using the recurrent relations (13), find $\alpha_k(\xi)$, $\delta_k(\xi)$, $\beta_k(\xi)$ and $\gamma_k(\xi)$ of the desired k layer and express the stress and displacement transformants using formulas (12).

6) The inverse integral Fourier transform (5) is applied to the obtained stress and displacement transforms of the layers.

Discussion. Let us make some comments on the formulated algorithm and its individual stages. Note that the knowledge of the compliance functions significantly reduces the amount of calculations, in particular, formula (15) allows us to halve the number of functions that determine the stress-strain state of each layer of the base. The basic functions for the first basic boundary value problem are $\alpha_k(\xi), \delta_k(\xi)$. Note that the compliance functions $A_k(p), A_{\tau k}(p), B_k(p), B_{\tau k}(p)$ can be determined for each layer at the first stage of the practical implementation of the algorithm.

The functions $\alpha_1(\xi), \delta_1(\xi)$ are calculated exactly in some cases only when given an analytical solution. In general, the calculation of these functions is assumed to be performed using approximate numerical integration formulas.

Given the known compliance functions of a multilayer base, it is sufficient to know two functions $\alpha_k(\xi), \delta_k(\xi)$ to determine the stress-strain state of the k layer of the base, since $\beta_k(\xi), \gamma_k(\xi)$ are determined using (15).

The key point in the implementation of the algorithm is the calculation of integrals when applying the inverse Fourier integral transform (5) to the stress and displacement transforms. These integrals are supposed to be calculated using quadrature formulas of the highest degree of accuracy.

It is worth noting that the formulated algorithm with the necessary changes can be applied to solving the second main boundary value problem for multilayer foundations with orthotropic layers.

Conclusions. The paper proposes an algorithm for analytical solution of the first basic boundary value problem of the plane elasticity theory for a multilayer base with orthotropic layers, which takes into account the material features. The considered approach can be used for numerical analysis of the stress-strain state in each layer of the base.

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