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FRACTAL IMAGE ANALYSIS OF WAVELET SPECTRA OF LINEAR FREQUENCY MODULATED SIGNALS

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Key words: *continuous wavelet transform (CWT) coefficient series, Hurst coefficient, fractal analysis, flat images of wavelet coefficient series, convolutional neural networks (CNNs), image augmentation.*

The article deals with the fractal analysis of images of continuous wavelet spectra of radar signals with linear frequency modulation, taking into account the level of additive Gaussian noise and the frequency modulation coefficient. As a result of the analysis, the numerical value of the level of Gaussian noise additively added to the signal above which the identification of the terrain image or signal spectrum is impossible to be obtained. The Mexican Hat wavelet function, which provides the maximum range of fractal dimensionality variation with noise variation, is determined. Methods of threshold detection by proximity of series of scaled wavelet noise and signal scales are investigated. An example of the Shapiro-Wilk normality test has shown the inefficiency of using statistical methods to determine the noise threshold of the series of wavelet coefficients that form the images of spectra. Two methods of detecting the noise threshold versus global scalograms and autocoherence of the signal and noise transformed into wavelet coefficient series are considered. The autocoherence method is more efficient due to the availability of numerical values. For the identified thresholds of two frequency modulation signals, with and without additional amplitude modulation, spectrum images are obtained and maxima of fractal dimension at the noise threshold boundaries are determined. By numerical values of maxima it is suggested to identify spectra by noise threshold for neural networks, for example, for preparation of a set of recognizable images.

ФРАКТАЛЬНИЙ АНАЛІЗ ЗОБРАЖЕНЬ ВЕЙВЛЕТ-СПЕКТРІВ СИГНАЛІВ ІЗ ЛІНІЙНОЮ ЧАСТОТНОЮ МОДУЛЯЦІЄЮ

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Ключові слова: *ряди коефіцієнтів безперервного вейвлет-перетворення (CWT), коефіцієнт Херста, фрактальний аналіз, плоскі зображення рядів вейвлет-коефіцієнтів, згорткові нейронні мережі (CNN), аугментація зображень.*

У статті проведено фрактальний аналіз зображень безперервних вейвлет-спектрів радіолокаційних сигналів із лінійною частотною модуляцією з урахуванням рівня адитивного Гаусівського шуму та коефіцієнта частотної модуляції. Отримано чисельне значення рівня аддитивно доданого до сигналу Гаусівського шуму, вище рівня якого ідентифікація зображення місцевості або спектру сигналу неможлива. Також визначено вейвлет-функцію «Мексиканський капелюх», котра забезпечує максимальний діапазон зміни фрактальної розмірності при зміні шуму. Приклад тесту нормальності Шапіра-Вилка показав неефективність використання статистичних методів визначення шумового порога рядів вейвлет-коэффициентов, які утворюють зображення спектрів. Розглянуто два способи виявлення шумового порогу порівняно із глобальними скалограмами й автокогерентності перетвореного на ряди вейвлет-коефіцієнтів сигналу та шуму. Найбільш ефективним у зв'язку з наявністю числових значень є метод автокогерентності. Для виявлених порогів двох сигналів частотної модуляції, з додатковою амплітудною модуляцією та без, отримано зображення спектрів і визначено максимуми фрактальної розмірності на межах шумового порога. За числовими значеннями максимумів запропоновано ідентифікувати спектри за рівнем шуму для нейронних мереж, наприклад, для підготовки набору зображень, що розпізнаються.

Introduction. Signal distorted by noise and maximum critical interference is processed using computer technology. An example is the distortion of medical and ultrasound images, localization trajectories, and visualizations of an object at a distance [1]. Since the image is a data set, it is very important to reconstruct the original signal to maximize the amount of data. We need to take into account that interference can occur during both signal transmission and reception.

A large number of different ways of filtering the signal from noise have been described in the literature. Linear and nonlinear filtering methods [2] such as Gaussian filter, median filter, mean filter and Wiener filter [1; 3] are widely used for noise reduction. A number of works have proposed algorithms for identification of non-Gaussian noise using various adaptive filters, such as the Kalman filter [4]. Common to these methods of information signal processing is filtering by noise removal and restoration of the useful signal.

The most complex, in our opinion, is the task of signal processing of radar detection systems. The task combines both accurate processing of information from huge amounts of data with visualization of the result, and processing of highly noisy signals [5]. The complexity of processing such signals is directly related to the a priori uncertainty about the probabilities of occurrence of detectable signals [6]. However, this area of technology is characterized by the fact that it often involves signal processing in situations where the share of noise in the signal may significantly exceed the information component. This condition cannot be ignored when developing methodological approaches to new methods of signal processing.

In addition, it should be noted that the efficiency of radar systems depends directly on the signal detection range [6]. This puts an additional requirement to the applied information technologies of signal processing. At the same time, the increase in resolution is achieved by reducing the pulse duration. This leads to a decrease in signal power and, consequently, in the detection range. This problem is a key issue in the use of information processing techniques in military radar systems to monitor enemy behavior [7] on the battlefield.

In recent years, information technology specialists all over the world actively pay special attention to the development of radar techniques based on wavelet transform for recognizing pulse modulation features. This is confirmed by the active implementation of innovations in this technology [7].

Literature Review. The use of linear frequency modulated signals is a compromise solution. The factor that affects the detection range is the modulation factor. The authors of the publication [2] propose a method of detecting a signal with linear frequency modulation by calculating the fractal dimension of linear spectrograms using the cell coverage method. This method does not reflect the peculiarities of signal modulation due to the limited time resolution of the Fourier spectrum.

The authors of [3] proposed to solve the problem of identification of a wide spectrum of signals by mathematical processing of discrete wavelet coefficients with subsequent filtering from noise [4–6] and construction of the correlation matrix of the obtained series of discrete wavelet coefficients [7].

In publication [8], an algorithm based on the use of continuous wavelet transform coefficients and higher order statistics in determining the features of selected signals was proposed. The principal component method was applied to reduce dimensionality. An artificial neural network was used as a classifier [8].

The continuous wavelet transform method has the ability to visualize the results. Such a representation is outlined in [9], which is devoted to the estimation of the Hurst coefficient [10] from the slope of the power spectrum based on the wavelet transform. The estimation of the Hurst coefficient for a volumetric data set allows us to identify belonging to certain patterns (trends or comparison signals) in the data set. However, the transformation of the input signal by accumulation from the mean does not allow to display individual features of the signal.

The use of fractal dimensionality is investigated in the monograph by G. Schuster, "Deterministic Chaos. Introduction" [11] using the correlation dimension $\ln C(\varepsilon)/\ln(\varepsilon)$ (where $C(\varepsilon)$ is a correlation integral) on the slopes of graphs $\ln C(\varepsilon)$ from $\ln(\varepsilon)$ for different noise levels at which, there are observed kinks on scales corresponding to noise levels below which the slopes are approximately equal.

However, at the time of publication of this fundamental work, the theory of the continuous wavelet transform was not yet sufficiently developed, and the author used the Fourier transform to analyze the spectrum [11 p.65]. Therefore, the type of noise interaction with the signal and its property (white or correlated) was not determined.

The solution to this question appeared later in [12 p.512]. Since the neighboring frequencies in the Fourier analysis for a time series of white noise are uncorrelated. This is no longer relevant to wavelet analysis. The correlation between CWT of two different scales s1 and s2 at different moments of time t1 and t2 is defined as:

$$C(s_1, s_2, t_1, t_2) = (W(s_1, t_1)W(s_2, t_2).$$

In the presence of uncorrelated white noise, the correlation between the scaling factors may change the nature of the noise effect on the fractal dimension, especially in the case of frequency modulation of the signal, as it may lead to the appearance of additional spikes. The main change is that the known statistical methods for determining the noise threshold for Fourier analysis, are no longer effective for wavelet reconstruction of image matrices.

Therefore, the computational experiment is relevant and timely, especially in connection with the use of wavelet transforms for the development of data set for neural networks

Determination of the dependence of the fractal dimensionality of the power of the wavelet spectrum of a signal on the noise level, frequency modulation coefficient and wavelet function

We will determine the wavelet coefficients from the known formula for the continuous local wavelet spectrum [3-5]:

$$W_{(a,b)} = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\psi\left(\frac{t-b}{a}\right)dt, \qquad (1)$$

where: *x(t)* is a signal with random component; is a basic wavelet from the list 'cmor11.5', 'cgau5', 'cgau1', 'shan0.5-2', 'morl'; $a \neq 0$ is a scale parameter; $b \geq 0$ is a shift parameter.

The data under study is discrete, so we write formula (1) in the form, selecting two arrays for scales *coeffs* for shifts *fred*:

$$coeffs, fred = \frac{\Delta t}{\sqrt{a}} \sum_{i=0}^{N-2} x(t_i)\psi\left(\frac{t_i-b}{a}\right). \qquad (2)$$

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When analyzing images of spectra, scale is of more interest than shift. Therefore, it is necessary to eliminate the dependence on the shift *b*, obtaining a representative amplitude of the scale inhomogeneity *coeffs* for *fred* shifts.

Obtaining an image of the power spectrum using the wavelet transform is to use their absolute values *abs(coeffs)* rather than the squares of the wavelet coefficients.

We will use the ratio for a signal with linear frequency modulation:

$$x(t_i) = anp\cos\left(2\pi f_0 t_i + \pi\beta t_i^2\right) + \eta_i, \qquad (3)$$

where *anp* is a signal amplitude; *f0* is an initial frequency value; *β* is a linear frequency modulation coefficient; *η* is an uncorrelated Gaussian noise with zero mathematical expectation.

The estimation of the noise influence is obtained from the relation:

$$SNR = 10\lg\frac{\sum \bar{x}(t_i)^2}{\sum[x(t_i) - \bar{x}(t_i)]^2}, \qquad (4)$$

where: $\bar{x}(t_i) = anp\cos\left(2\pi f_0 t_i + \pi\beta t_i^2\right)$.

To calculate the fractal dimension D=2-H through the Hurst coefficient (H) for a range of absolute values of scaling wavelet coefficients, we use the following algorithm. We create a range of delay values, calculate an array of delayed differences and after linear approximation obtain the Hurst exponent. We will investigate the change of fractal dimension of signals in the frequency range from -14 to 14 dB, for frequency modulation coefficients β=128, 256, 512 and initial frequency f0=50 MNz of five continuous wavelets. To find the wavelet that provides the maximum range of variation of the Hurst coefficient (H), we plot the graph.

It can be seen from the above graphs that the dependence of fractal dimensionality on noise power is nonlinear and decreases with decreasing noise. The range of variation of the dimensionality depends on the selected wavelet and is maximal except for its value at $\beta = 256$, which indicates its influence on the distribution.

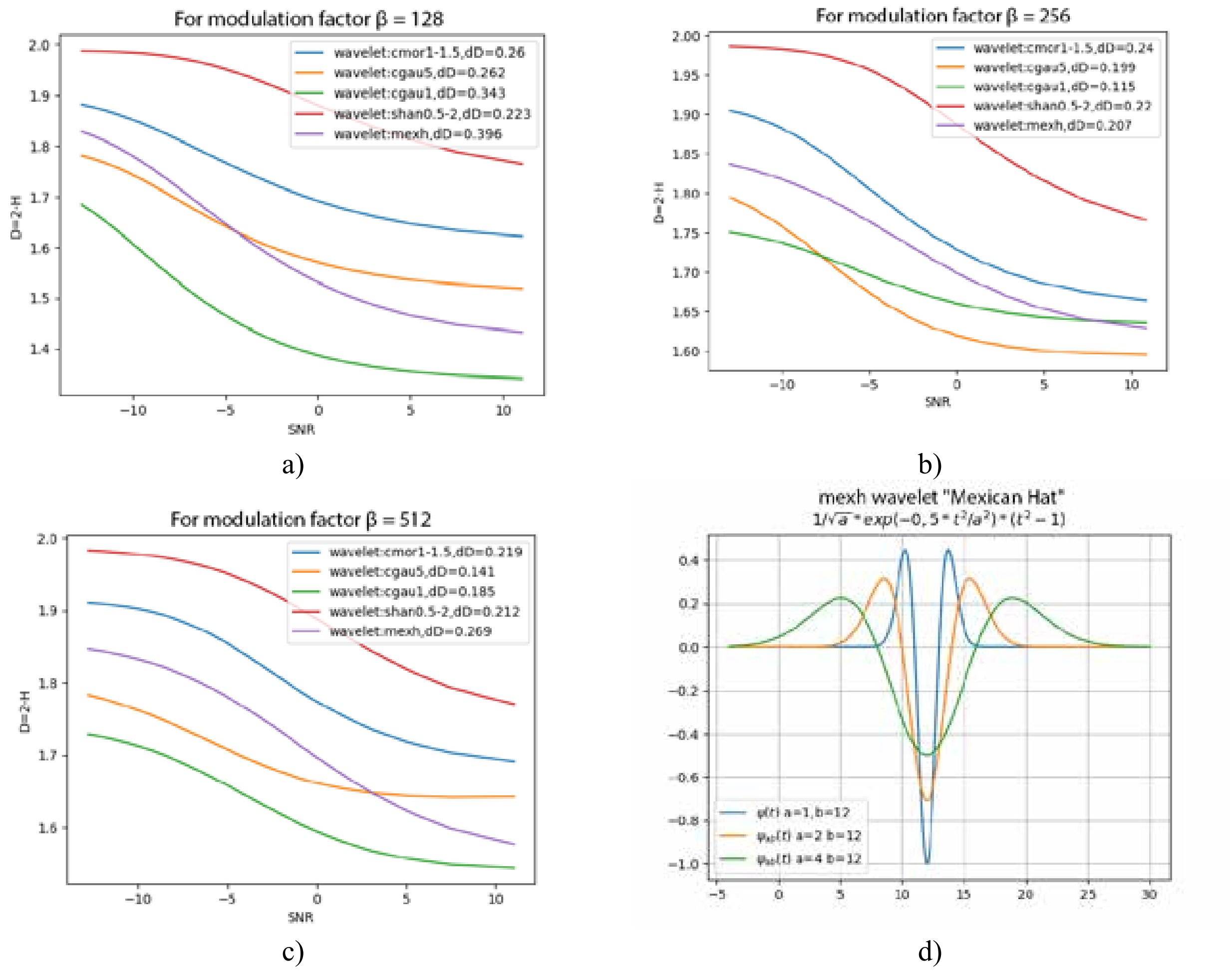


Fig. 1. Fractal dimension of the spectrum and time domain of the main wavelet: a) for modulation coefficient b) c) c) mexh wavelet

Analyzing the applicability of statistical methods for determining the noise threshold for series of scaling wavelet coefficients of the spectrum image on the example of the Shapiro-Wilk test

For signal (3) in the noise range from -14 to 14 dB, the noise threshold by the Shapiro-Wilk test is determined as shown in the following figure:

At uncorrelated noise and correlation of scaling wavelet coefficients by the ratio [12]:

$$C(s_1,s_2,t_1,t_2)=(W(s_1,t_1)W(s_2,t_2)\,. \qquad (5)$$

The value of noise threshold for continuous wavelet spectra cannot be determined because the statistics of detection p over the entire range of noise is zero as shown in the graph:

Statistical methods of noise threshold detection using the example of Shapiro-Wilik test are unsuitable for noise threshold detection on wavelet spectrum images.

Method of detected noise threshold by comparison of global wavelet scalograms

The local power spectrum is determined by relation (1) and the global one by relation [13]:

$$E_w(\mathrm{a})=\int_{-\infty}^{\infty}W^2_{(a,b)}db=\int_{-\infty}^{\infty}E_w(\mathrm{a,b})db\,. \qquad (6)$$

Ratio (6) is called the scalogram or variance of the wavelet transform. The scalogram $E_w(\mathrm{a})$ corresponds to the power spectrum smoothed at each scale by the Fourier spectrum of the analyzing wavelet. This allows us to obtain more localized power information necessary to detect the noise threshold.

We will compare the scalogram of the noise generated in the scale of the signal with the scalogram of the deterministic signal with noise, achieving a change in the noise level of equality of these scalograms.

In Fig. 4 on the mashtabograms, the white spots reflect the noise. The comparison results reflect the scalograms, in which the Fourier power spectrum and the global wavelet spectrum with 95% confidence level are given.

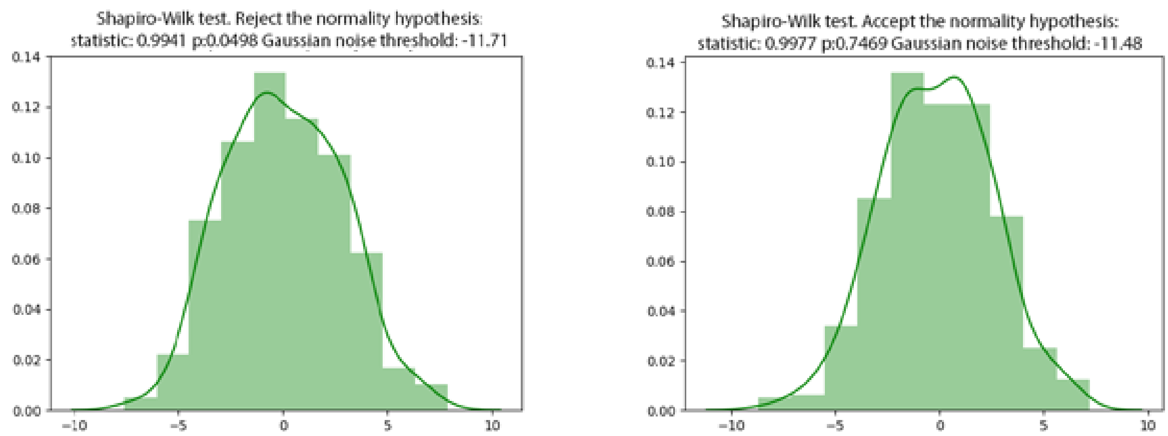
A method for detecting the noise threshold from the autocoherence of a number of wavelet coefficients of the signal itself

The crossed wavelet spectrum discussed above for testing the significance of the relationship between two processes, one of which is Gaussian white noise, is difficult to test. Instead, it is better to apply autocoherence according to the series of scaling wavelet coefficients (2). And calculate the derivatives using the correlation distance between the series for signal and noise by the ratio:

$$\begin{cases} S_x=\dfrac{\Delta t}{\sqrt{a}}\displaystyle\sum_{i=0}^{N-2}x(t_i)\psi\left(\frac{t_i-b}{a}\right),\\ S_y=\dfrac{\Delta t}{\sqrt{a}}\displaystyle\sum_{i=0}^{N-2}\frac{dx(t_i)}{dt_i}\psi\left(\frac{t_i-b}{a}\right).\\ k^2=\dfrac{|S_{xy}|^2}{|S_x|^2|S_y|^2}.\end{cases} \qquad (7)$$

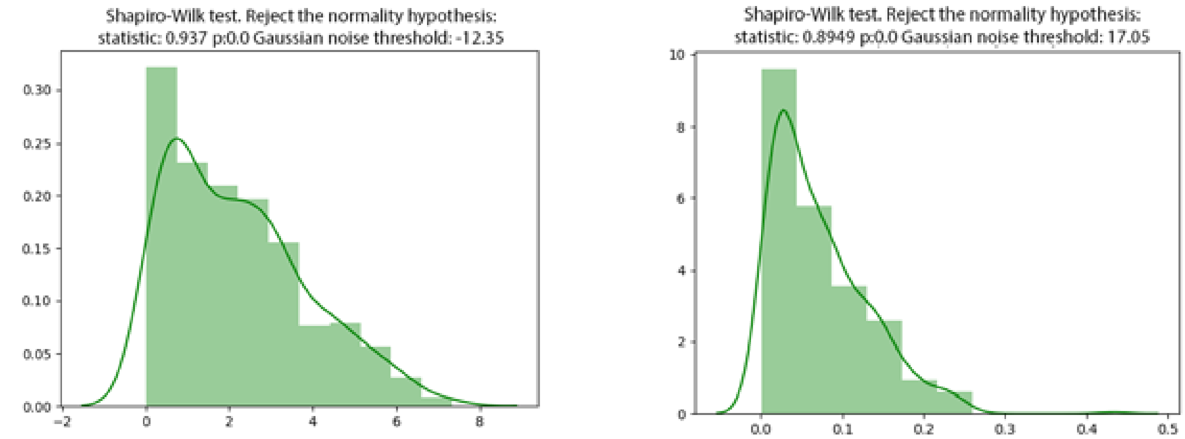
It should be noted that the system of equations (7) is a rational modification of the well-known wavelet coherence system [14], but it is based not on the comparison of two different series, but on the comparison of the series itself with its derivative in the time domain.

For reproducibility of the results of comparison with noise it is necessary to ensure equality of scales and the same wavelets used for decomposition. In addition, the size of the image itself (extent) [15] and



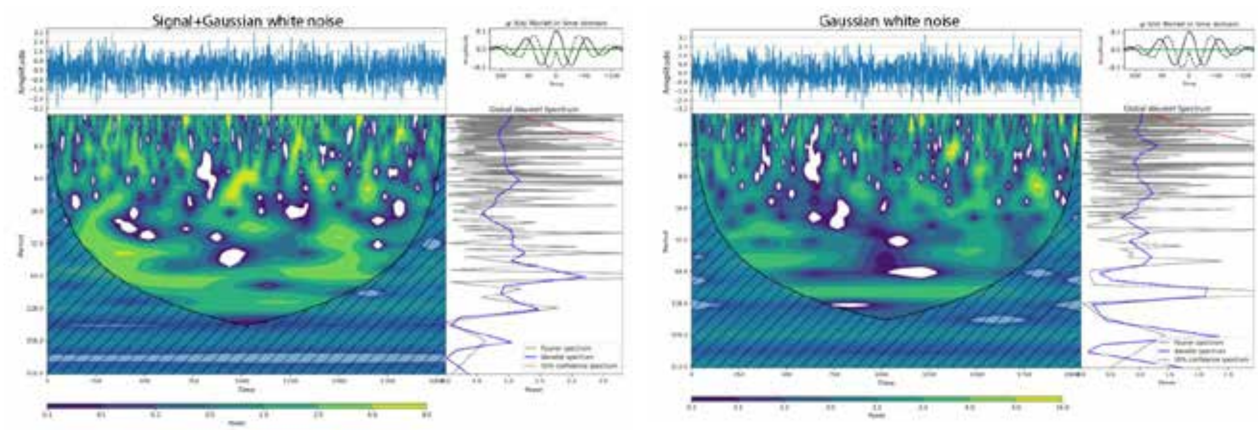
a) b)

Fig. 2. Detection of noise threshold for linearly frequency modulated signal by relation (3):
a) – distribution for noise threshold; b) – distribution up to threshold per step $\sigma=0{,}1$ downward σ



a) b)

Fig. 3. Failure of the statistical test for the wavelet spectrum: a) – distribution for 12.35 dB noise; b) – distribution for 17.5 dB noise



a) b)

Fig. 4. Wavelet scalograms: a) for a signal with noise is 6.95 dB for $\beta = 512$; b) noise is 7.3 dB

the palette (cmap, aspect) should be the same during visualization.

Let's build autocoherence for signals and noise on one graph by changing the noise level in the range from -14 to 14 dB. Let's find the intersection points of noise and signal series for maxh ("Mexican Hat") wavelet.

Analysis of wavelet spectrum autocoherence from the signal (3) for comparison is necessary together with the ratio taking into account the amplitude modulation of the half-sine wavelet:

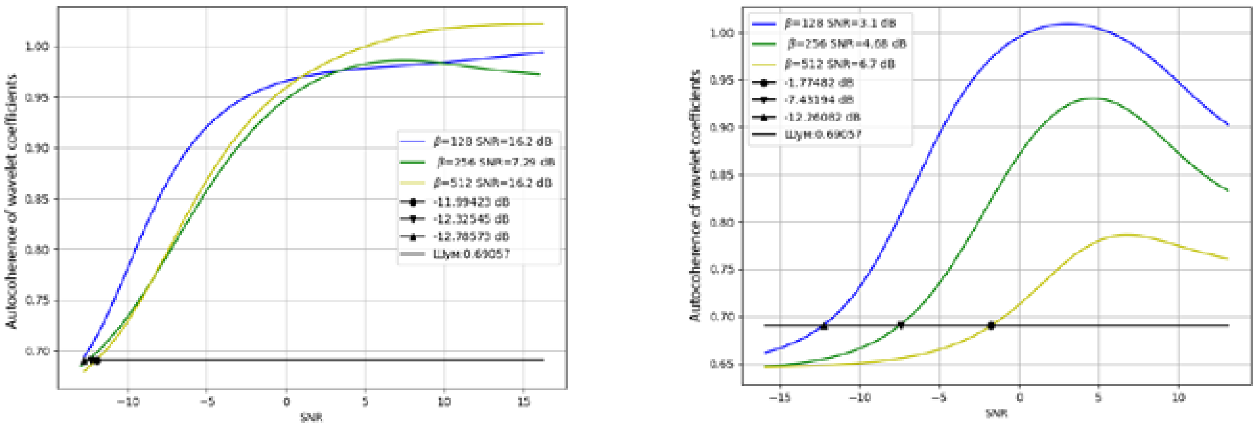
$$x(t_i) = anp\cos\left(2\pi f_0 t_i + \pi\beta t_i^2\right)\sin\left(\pi t\right) + \eta_i . \quad (8)$$

Using the fractal dimensionality of wavelet spectrum images to prepare DataSet neural networks with taking account of noise thresholding

Preparation of images in the form of scalograms significantly increases the efficiency of neural networks for image recognition. In this case, to form a data set it is necessary to form a set of signals with different noise levels and at the same time be sure that the signal carries information and not chaos.

For this purpose, let's transfer the already defined noise threshold to the dependence of fractal dimension on noise for spectra images. For this purpose, we need to make sure that the character of changes in fractal dimensionality is close to the changes for series of scaling wavelet coefficients.

In [16, 17], the following cell covering method is proposed to calculate the fractal dimensionality of images. For our problem, the spectrogram images are reduced to binary so that a pixel is considered filled but not white. According to the cell coverage method,



a) b)

Fig. 5. Determination of noise threshold using autocoherence of continuous wavelet spectrum for wavelet mexh and scales=range(1,128): a) for frequency modulated signal by relation (3); b) for signal by relation (8).It should be noted that the noise threshold in both plots is about 0.7, which corresponds to the noise power. For a frequency modulated signal the level of noise threshold does not depend on the modulation coefficient

the fractal dimensionality D can be defined as a linear regression coefficient from the relation:

$$log\frac{1}{s}, -logC(s) = 0\,, \tag{8}$$

where*:* s is the size of the side of the image blocks in pixels; *C(s)* is the number of image blocks of size s that have at least one filled pixel.

Let us define the fractal dimensionality of the images of the considered spectra:

The visually undetectable image change (Fig. 6) is identified numerically by the maximum of fractal dimensionality with sensitivity (1.6122-1.6106)/(-13.3 +11.6)=-0.0016/1.7. Change in fractal dimension (Fig. 7) with sensitivity (1.6116-1.5979)/(-11.7+8.3)=-0.0137/.4. The changes in the third and fourth digits do not seem significant, however, such a change is stable and in some cases may be the only way to select images in the data set for neural network. Such a task is relevant, because the insufficient number of images associated with their limited selection strongly reduces their recognition in neural networks [17-20].

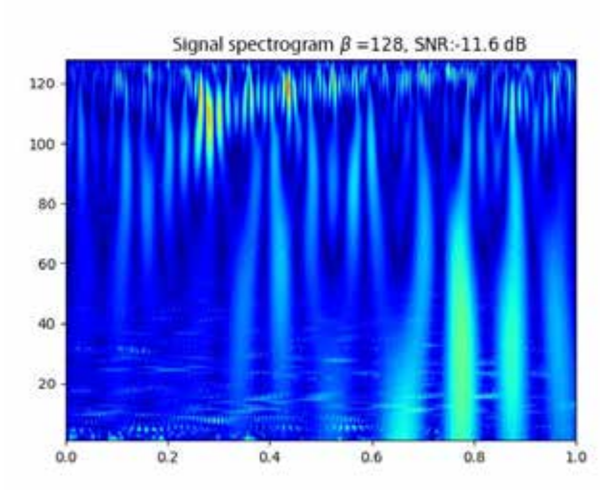
Discussion. The need for greater accuracy of information processing pushes us to search for new methods of noise filtering. The work performed allows to prepare any image for noise cleaning, which is important for many areas of human activity (machine learning, computer vision, aerial photography, etc.).

In the article, for the first time, the autocoherence wavelet of the noisy signal and noise is compared to determine the level of noise in the signal. It is shown that the autocoherence of noise depends only on the law of its distribution and continuous wavelet, and the autocoherence is constant over the whole range of noise power variation, which allows to use noise augmentation when preparing data for CNN.

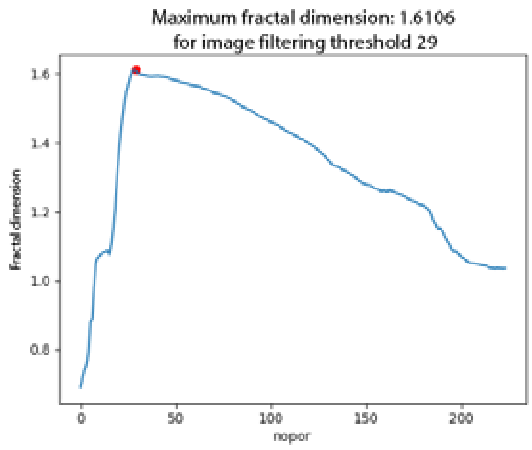
Fractal time series analysis considers the behavior of the system not only at a given moment but also its prehistory. Wavelet analysis is applicable to non-stationary data processing. It provides local high-frequency and global large-scale information about an object. It also allows us to judge at what point in time certain signal components appeared. In this article, for the first time, a noise threshold is found, which improves the detection accuracy.

The question of the feasibility of using fractal analysis together with continuous wavelet analysis of images when preparing a data set for CNN remains open. Since convolutional neural networks (CNNs) are used to identify images, terrain, or signal spectra, in which Gaussian noise is added to the image data set to improve recognition accuracy [3] and when the level of added noise is exceeded, it can have the opposite effect when the image signal is converted into noise.

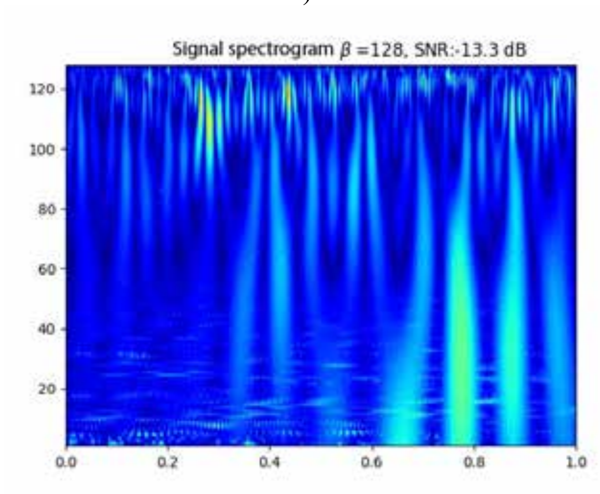
Conclusions. Dependences of the fractal dimensionality of wavelet spectra images of a frequency modulated signal on the noise level and modulation coefficient are obtained. The Mexican Hat wavelet function, which provides the maximum range of fractal dimensionality variation with noise change, is determined. Methods of threshold detection by proximity of series of scaled wavelet noise and signal are investigated. On the example of the Shapiro-Wilk normality test the inefficiency of using statistical methods to determine the noise threshold of wavelet coefficient series that form images of spec-



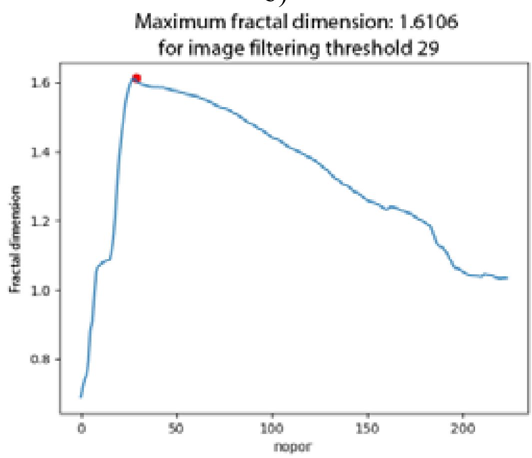
a)



b)



c)



d)

Fig. 6. Spectrum images and maxima of fractal dimensionality for transition through noise threshold for signal (3): a) wavelet spectrogram image for noise up to noise threshold; b) maximum of fractal dimensionality for spectrogram image; c) spectrogram image for noise above noise threshold; d) maximum of fractal dimensionality for spectrogram image

tra is shown. Two methods for detecting the noise threshold are considered, comparing global scalograms and autocoherence of the signal and noise transformed into wavelet coefficient series. The autocoherence method is more efficient due to the availability of numerical values. For the identified thresholds for two frequency modulation signals, with additionally amplitude modulation and without. Spectra images are obtained and maxima of fractal dimensionality at the noise threshold boundaries are identified. By numerical values of maxima it is proposed to identify spectra by noise level, for example, for preparation of a set of recognizable images, for neural networks.

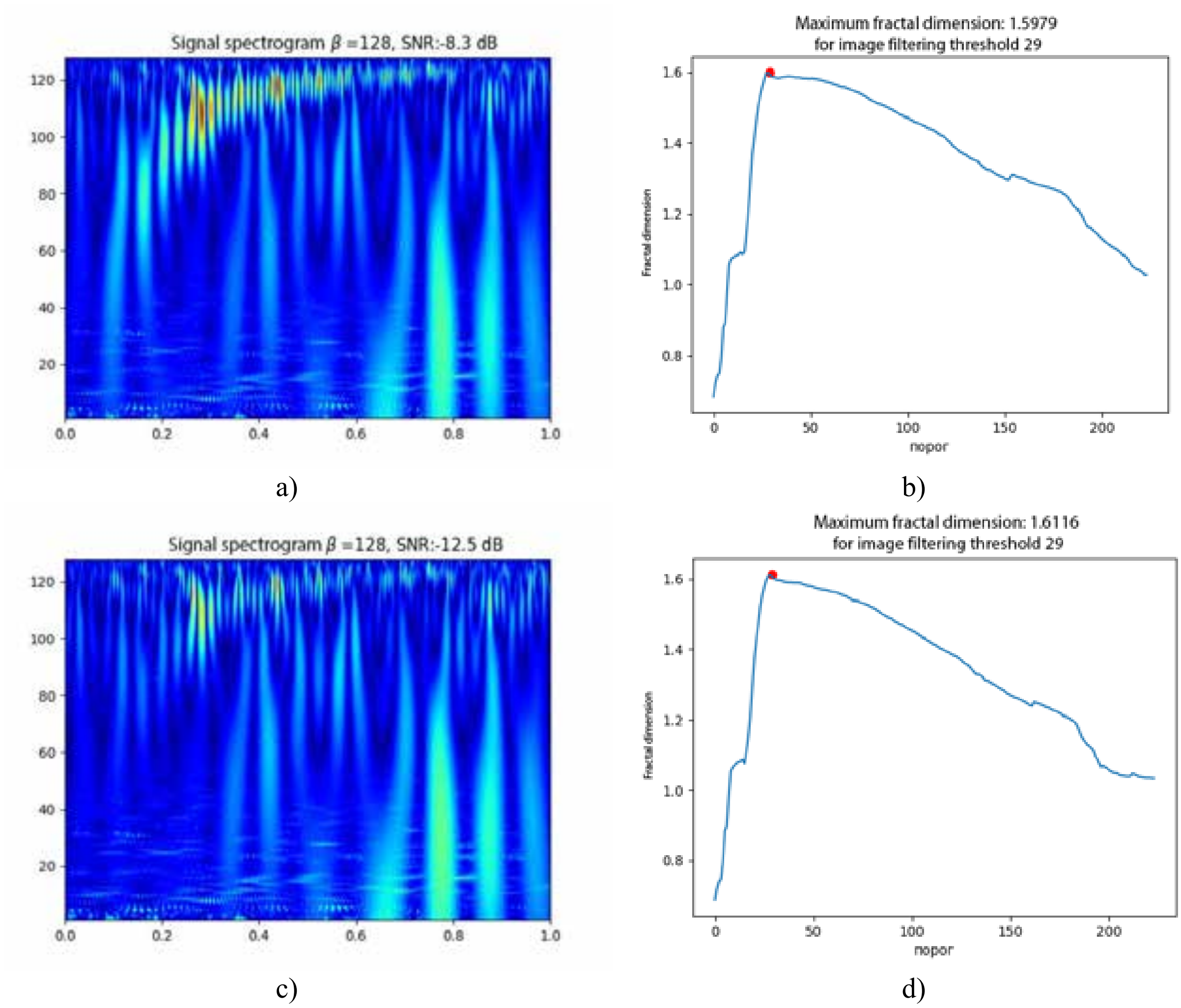


Fig. 7. Spectrum images and maxima of fractal dimensionality for transition through noise threshold for signal (8): a) wavelet spectrogram image for noise up to noise threshold; b) maximum of fractal dimensionality for spectrogram image; c) spectrogram image for noise above noise threshold; d) maximum of fractal dimensionality for spectrogram image

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