FINANCE AND MONEY TURNOVER

UDC 368.03:303.09:005.931.11 https://doi.org/10.26661/2414-0287-2020-4-48-18

INSURANCE COMPANIES INSOLVENCY PROBABILITY MODELLING

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Key words:
insurance organization, dynamic model, probability of bankruptcy, inflation, rate of interest.

The article is dedicated to problems of developing scientific and methodological foundations as well as creating an dynamic mathematical model of insurance company solvency (bankruptcy probability) considering interest rate and inflation ratio.
The study used the general scientific and special methods such as: the method of critical analysis, scientific abstraction and generalization of scientific expertise of recent theoretical studies, system-integrated approach, method of dynamic mathematical modeling.
Elaboration of dynamic mathematical model of bankruptcy probability modelling considering inflation (as inflation has negative impact on all aspects of insurance business including insurance reserves) and rate of interest. The peculiarities of insurance companies investment activity have been defined. The estimation of insurance premium that ensures abequate insurance fund value formation, i.e. insurance company solvency formation has been performed. Insurance tariff and supplement value correspondent to defined probability of insurance company bankruptcy have been defined.
Methodological approaches of insurance companies solvency (bankruptcy probability) modelling were further developed. The dynamic mathematical model of bankruptcy probability considering inflation and rate of interest has been proposed.
Theoretical study was developed to the level of specific techniques and suggestions for improvement of the estimation and prognozing of insurance companies solvency and could be used in strategic, current and operational planning. A comprehensive methodology of supplement estimation allows to respond to the changing market situation by changing the values of insurance tariffs.

МОДЕЛЮВАННЯ ЙМОВІРНОСТІ НЕРОЗОРЕННЯ СТРАХОВОЇ ОРГАНІЗАЦІЇ

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Ключові слова:
страхова організація, динамічна модель, ймовірність нерозорення, інфляція, норма доходності.

Метою роботи є побудова динамічної математичної моделі ймовірності розорення страхової організації, яку можна вважати критерієм платоспроможності, із врахуванням норми доходності та показника інфляції.
У дослідженні було використано загальнонаукові і спеціальні методи дослідження: метод критичного аналізу, наукової абстракції та узагальнення наукового досвіду сучасних теоретичних досліджень, системно-комплексний підхід та метод динамічного математичного моделювання.
Побудова динамічної математичної моделі нерозорення страхової організації із врахуванням рівня інфляції (оскільки інфляція негативно впливає на всі аспекти страхової справи, в тому числі і на стан страхових резервів) та норми доходності інвестиційних вкладень. Визначено особливості інвестиційної діяльності страхових компаній. Проведено оцінку величини страхового внеску, який забезпечує формування достатньої величини страхового фонду, тобто платоспроможність страхової організації. Відповідно визначено страховий тариф та величину навантаження до страхового тарифу, яке відповідає заданій ймовірності розорення страховика.
Дістали подальшого розвитку методичні підходи до моделювання ймовірності неплатоспроможності (ймовірності нерозорення) страхової організації. Запропоновано динамічну математичну модель ймовірності нерозорення, яка враховує показник інфляції та норму доходності.

Запропонована модель оцінки ймовірності нерозорення страхової організації дозволяє оцінити та прогнозувати платоспроможність страховика та може бути використана при стратегічному, поточному та оперативному плануванні діяльності страхової організації. Отримана залежність навантаження до страхового тарифу дозволяє страховій компанії більш оперативно реагувати на можливі зміни ринкової кон'юнктури шляхом зміни страхового тарифу.

Problem statement

In the context of the economic crisis, when insurance companies are forced to reduce their activities the solvency of insurers comes to the fore. This allows to ensure a more balanced approach for tariff rate fornmation and investment activity implementation.

Insolvency risk determination is one of the most important tasks for both an insurance organization and national insurance. Investment policy and formation of investment portfolio plays an important role in ensuring solvency of insurers.

In addition, it can be noted that most of the proposed solvency models are static, but dynamic insurance theory describes the state of the insurance organization more adequately than static models.

Recent research and publication analysis

The problem of insurance companies solvency management in the context of ensuring their financial stability has been investigated by domestic and foreign scientists. Thus, I. Nenno [1, p. 261] considers solvency as the main sign of financial stability of insurance companies, the specifics of which are manifested in the peculiarities of the formation of obligations and resources for their implementation.

The Kramer-Lundberg and other stochastic risky models are classic approaches to solving the problem of ruining the insurance company. The methods are to construct the integral or integro-differential equations [1-4] for the insolvency probability.

Among the domestic and foreign scientists, whose works are dedicated to the calculation of the insolvency probability of the insurance company A. V. Melnikov [2] A. V. Boykova [3, p. 550-552], O. M. Androschuk [4, p. 1448-1450] should be noted.. In the work of O. V. Zhumik, Y. A. Stadnyk [5, p. 150] methods of actuarial mathematics were used to estimate the probability of insurance company bankruptcy.

The financial sustainability of insurance organizations is significantly influenced by the level of inflation. Inflation distorts economic calculations, makes uncertain development prospects, increases investment risks. However, in most solvency models, the inflation rate is not used.

The purpose of the article

The purpose of the article is to develop scientific and methodological foundations for creating an dynamic mathematical model of insurance company solvency (bankruptcy probability) considering interest rate and inflation ratio.

The main material representation

By probability of collapse, we mean the quantification of the possibility of an event, in which the insurance loan at a certain point in time will be greater than the sum of insurance organization reserves and collected insurance premiums. In the collective risk model, the probability of ruin can be considered either at a certain point in time, or for infinite time interval. The ruin probability determinftion is one of the key tasks of classical risk theory. [5-7].

The assessment of the ruin probability not just by missing bankruptcy, but also its non-admission, is the basis for the construction of the basic concept of ensuring insurance company solvency. Quantitative assessment of the non-ruin probability, on the other hand, allows to find the optimal (rational) amount of insurance premium sufficient to form an adequate insurance fund.

The ability to predict the insurance organization insolvency risk with the maximum degree of probability allows to work out plans and forecasts a strategy to counteract risk. The variety of forms of risk manifestation, frequency and severity of the consequences of its implementation cause the need for an in-depth analysis of the risk and economic and mathematical justification of the financial policy of the insurance company. The use of economic and mathematical methods allows to obtain more bottomed and reliable assessments of the main characteristics of insurance organization solvency.

Dynamic insurance theory describes the state of the insurance organization more adequately than static models. The main ideas of ruin probability dynamic modeling and the state of the insurance company can be defined as follows [6]:

– the dynamic model takes into account the time factor t and the processes of contributions and claims are described by a random process;

– the value of the current capital of the insurance organization depends on the time factor t.

– the ruin probability is the characteristic of insurance organization solvency.

A generally accepted measure of risk in insurance is the probability of insurance organization bankruptcy (risk of insolvency) during the time T, which can be defined as [6]:

$$1 - P(\{\omega : R(t) \geq 0, t \in (0, T)\}). \quad (1)$$

Suppose that the number of possible policyholders' claims N is unknown. It should be noted that in a dynamic collective risk model Time-dependent number N=N(t) of claims is a random process that determines the number of claims filed during the period [0.t]. As noted in the paper [6], the collective risk model is more realistic and provides more risk management opportunities for the insurance company. Taking into account that number of claims X_i is positive and independent of N the risk model can be presented this way:

$$X = \sum_{i=1}^{N} X_i. \quad (2)$$

Here, $\varphi(x,k) = P(\{\omega: R(j) \geq 0, j = 0,1, \dots . k\})$ determines the solvency probability i.e. the insurance organization solvency at a certain time interval [0,k], and the function $\varphi(x) = \lim_{k\to\infty} \varphi(x,k)$ determines this probability at an infinite interval [0,∞]. Definition of these analytical functions allows to evaluate the solvency of the insurance company.

Inflation affects all aspects of insurance including the insurance reserves, and this impact is quite complex and multifaceted. The main areas of influence are:

1) Impact on the consistency of insurance reserves with obligations assumed by the insurer. It is noted [8] that in determining the extent of damages under individual contracts, the impact of inflation varies by type of insurance, which in turn can change the value structure of the portfolio;
2) the impact of inflation depends on the duration of the insurer's liabilities. With minor liabilities, such as in short-term insurance, the insurer has the ability to adapt insurance conditions to a real inflationary process;
3) inflation seriously affects the investment of insurance reserves and the structure of the insurer's reserves, as a result of this, the gap between the expected interest and the real one may present a serious problem for the insurance company, since insurers in one way or another are obliged to invest the share of assets in "safe" securities that have a predetermined discount rate. Increasing interest rates can cause both impairment of such papers and impairment of the portfolio as a whole;
4) inflation affects the investment income of the insurance organization as the basis for indexing liabilities, and the insurer's profit on investment transactions can be negative.

However, it should be noted that taking into account all aspects of the inflation impact could cause some technical difficulties. Against this backdrop, the annual retail price index is generally accepted as the inflation rate.

The deterministic treatment of inflation in the insurance organization solvency model has been proposed in the work [8]:

$$f(t) = \bar{f} + 0.5 \cdot A(t), \quad (3)$$

where f(t)=+0.5·A – the amount of inflation for the period of time (t), $\bar{f}$ – average inflation rate , A(t) – the amplitude of the GDP cycle.

An important issue in assessing solvency is the consideration of premium' inflation. It is necessary to take into account the impact of inflation on the financial condition of the company when calculating insurance tariffs, developing investment policy, planning, etc. To do so, insurers must forecast inflation and, on the basis of this forecast, and, on the basis of this forecast, make management decisions.

Thus, in order to determine the amount of premium inflation, the insurer has to identify past inflation and forecast its level for the future.

Insurers do not have the opportunity to estimate the amount of inflation for a certain period of time immediately after its expiration. Moreover, the insurer makes a decision not immediately, but a certain time after receiving reliable information about the inflation rate. Obviously the insurance organization should adjust the tariffs (listed with the new forecast value of inflation) with the insurance supervision bodies.

Thus, in practice after the end of the time period there is a certain time lag. Such approach determines inflation over a period of time (year) t equal to the value of inflation with an annual time lag:

$$f_{пр}(t+1) = f_{пр}(t-1-t_d), \quad (4)$$

where t_d - the number of years of delay in determining the actual inflation rate.

This method allows to determine the value of inflation premiums for the current year, but also this approach suggests that the duration of the settlement period may exceed the year.

In determining inflation for the short term, the insurer cannot be guidedonly by the result of the last year. For example, the value of premium inflation is equal to the arithmetic mean of inflation over the last five years:

$$f_{пр}(t+j) = \frac{\sum_{y=2}^{6} f(t-y)}{5}, \quad (5)$$

where j:0÷d, j – the forecast duration indicator, y – depth variable of the prediction base, d – the settlement period maximum length

It should be noted that this method is accurate, and the use of the mean eliminates forecasting errors caused by inflation rate fluctuations. Ite can be seen that the forecast of premium inflation is the same for all j at a fixed value of t. In order to get the total inflation rate for several years from the moment t , it is necessary to erect a value (1+f(j)) to a power j.

It is also necessary to introduce a variable that will mark total inflation, i.e. the value of retail prices increasing:

$$f_{сум}(t) = \prod_{j=1}^{t=1}[1 + f(t-j)], \quad t > 1.$$
$$f_{сум}(t) = 1 \quad \text{при}\ t = 1$$

Accordingly, it is convenient to use the value of total inflation rate while determining the total premium inflation:

$$f_{сум}(t) = \prod_{j=1}^{t=1} f(t-j) \cdot f_{сум}(t-t_d), \quad \text{при} \quad t > t_d$$
$$f_{сум}(t) = 1 \quad \text{при}\ t > t_d.$$

Let assess the inflation rate impact on the yield rate. The total inflation rate for n periods can be defined as:

$$f_{сум} = \prod_{1}^{n}(1 + f_i) - 1. \quad (6)$$

The mean annual inflation rate will be defined with the assumption that the intervals for determining the inflation rate are the same and apply the concept of the mean inflation rate over the period. With this approach, one can take into account that the inflation rate during the year is constant and will receive the mean annual inflation rate in the form of:

$$\bar{f} = n\sqrt{\prod_{1}^{n}(1 + f_i) - 1}. \quad (7)$$

Let assess the inflation impact on the annual interest rate. For the current annual interest rate of income in the context of inflation, we have:

$$r^* = \frac{r-f_s}{1+f_s}. \quad (8)$$

Thus, in further calculations we will take into account the interest rate with an amendment to the inflation rate (r*). For convenience let delete the sign *.

In a market economy insurance is an important factor in the development of investment activity. The investment potential of the insurance organization consists of two main parts – own capital and secured resources. As a result of sectoral specificities, the value of secured resources exceeds own capital. The activities of the insurance organization are based on the creation of cash funds which are received in the form of premiums and are temporarily held by the insurance company. The funds are then used to pay the contract claimsor become the income of the insurer. but before it becomes income of the insurer, these funds can be used as an investment source, or as other legally provided purposes.

The attractiveness of the insurer's reserves requires it to have a sound investment policy and to take investment risks into account in its investment choices. The insurer's ability to meet insurance obligations depends on the investment performance of the insurer: If the insurer's investment programme is not well thought out and the subject matter of the investment is unsatisfactory, the insurer may face bankruptcy.

In our opinion, the investment activity of insurance organizations has some peculiarities:

1. Insurers in developed countries are always in the top three big investors (banks, insurance companies and pension funds). The ratios of these three components may vary from country to country, but the composition remains unchanged.
2. Insurance organizations own two types of investment resources: own funds (which are not incidental in nature and are not directly related to the fulfilment of insurance obligations) and insurance reserves (which are incidental and directly related to the fulfilment of insurance obligations). It should be noted that each of these components has its own specificity in terms of their ability to invest.
3. The investment activities of insurance organizations are strictly regulated by the State, especially with regard to the investment of funds that represent insurance reserves.
4. For the insurance company, the investment is not a core activity, but it is an ancillary one, although for Life companies this is less significant. At the same time, insurance activities impose restrictions on investment activities.

Consider the risk model of the insurance company, which operates in a two-dimensional (B,S)-market by placing temporarily available funds in shares S (risky asset with profitability ρ_n) and a deposit account B (risk-free asset with a yield rate r) [6]:

$$\Delta B_n = r \cdot B_{n-1}, B_0 > 0;$$
$$\Delta S_n = \rho_n \cdot S_{n-1}, S_0 > 0, n \le N,$$

where $r \ge 0$ - the interest rate ($a<r<b$).

The level of profitability:

$$\rho_n = \begin{cases} b \ with\ probability\ p \in [0,1] \\ a\ with\ probability\ \ q = 1-p, \end{cases}$$
$$n = 1, \dots . N.$$

The insurance organization's risk model takes into account the rate of return on which, in turn, the inflation rate is affected. Determining inflation indicators is one of the difficult moments in the implementation of investment activity and assessment of solvency of the insurance organization. Let assume that an insurance organization with capital $x = R_0$ forms an investment portfolio (β1,ρ1) at a time n =0:

$$R_0 = \beta_1 \cdot B_0 + \gamma_1 \cdot S_0. \quad (9)$$

At a time n=1 capital value is equal to:

$$R_0 = \beta_1 \cdot B_1 + \gamma_2 \cdot S_1 + V - Z_1, \quad (10)$$

where V – amount of insurance premiums; Z1 – a positive random value, which represents the total amount of insureds' claims for payments during a certain period.

This capital is reinvested in the portfolio (β2,ρ2), so that the amount of capital is $R_2 = \beta_2 \cdot B_2 + \gamma_2 \cdot S_1$. Thus for the time n we have:

$$R_n = \beta_n \cdot B_n + \gamma_n \cdot S_n + V - Z_n, \quad (11)$$

where the sequence $\pi = (\beta_n, \gamma_n)n \ge 0$ represents an investment strategy and Z_n represents a positive random value that characterizes the total number of claims during the period {(n – 1); n}.

Let define the differential function of Z_n distribution as F(Zn) ≡ F(Z). Consequently, the capital dynamics of the insurance company is characterized by:

$$R_{n+1} = \beta_n \cdot B_{n+1} + \gamma_n \cdot S_{n+1} + V - Z_{n+1} = R_n \cdot (1+r) + \gamma_n \cdot S_n(\rho_{n+1} - r) + V - Z_{n+1}. \quad (12)$$

It has been shown [6] that the insolvency probability in general can be expressed by a formula:

$$\psi_{k+1}(x) = e^{-\mu[x(1+r)+V]} + \int_0^{x(1+r)+V} \psi_k(x(1+r) + V - y)\mu \cdot e^{-\mu y} dy, \quad (13)$$

де $\psi_1(x) = e^{-\mu[x(1+r)+V]}$.

Consider the placement of insurance organization funds both in risky and risk-free assets, the ratio of which in the insurance organization investment portfolio is α and suppose there is a constant ratio $\alpha_n \equiv \alpha$. In the case of exponential distribution, we get an estimation:

$$\alpha_n = \frac{\gamma_{n+1} Z_n}{R_n}. \quad (14)$$

It should be noted that γn+1 is the the number of assets names (S) that company buys after the insurance contracts conclusion for the total amount V and claims payment for an amount Z_n, and thus the amount of capital is R_n. The dynamics of the insurance organization's own capital can be described by the equation:

$$R_{n+1} = R_n \cdot (1+r) + \gamma_n \cdot S_n(\rho_{n+1} - r) + V - Z_{n+1}. \quad (15)$$

The probability of insurance organization insolvency can be determined as:

$$\varphi_1(R_0) = P(\omega: R_1 < 0) = 1 - pF_Z(R_0[1 + r + \alpha(b-r)] + V) - qF_Z(R_0[1 + r + \alpha(a-r)] + V). \quad (16)$$

ISSN Print 2414-0287
ISSN Online 2707-8426

In the case the claim flow is described by an exponential function F(y)=1−e−λy, we have:

$$\varphi_\infty(x) \le \varphi_1(x)\left[1 - \frac{r+p\alpha(b-r)+q\alpha(a-r)}{[r+\alpha(b-r)][r+\alpha(a-r)]}e^{-\lambda V}\right]^{-1}. \quad (17)$$

In the case of investment in risk-free assets, i.e. rα=0, and on the condition that e−λc, we have:

$$\varphi_\infty(x) \le \varphi_1(x)\left[1 - \frac{e^{-\lambda V}}{r}\right]^{-1}. \quad (18)$$

We will assess the amount of insurance premium sufficient to provide the insurance fund, which, in turn, allows to ensure the insurance organization solvency. The insurance company concludes N typical insurance contracts, and policyholders claims form streams with the same π(V). Then the intensity of the total flow of N customers' claims can be determined as $N\pi(V)$. Thus:

$$\int_d^K \pi(dy) = V_0$$

$$\int_d^K y\pi(dy) = V_1\,. \quad (19)$$

The insurance policy price is V>0, and the total revenues amount is VN (K – insurance company' capital at the beginning of the insurance period).

Conclusions

Using a stochastic approach, an integral equation of the probability of insolvency of an insurance company has been obtained, which can be used to estimate and predict the insolvency risk of insurance organizations. Further research focuses on the distribution of the main parameters of the integral equation of insolvencies of insurance organizations , in particular the rate of return or the measure of inflation , and the determination of their numerical characteristics. Using the random value conversion function, you can get a differential probability distribution function for insurance company insolvency.

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