ISSN Print 2786-5150
ISSN Online 2786-5169

ECONOMIC AND MATHEMATICAL MODELING AND INFORMATION TECHNOLOGIES IN ECONOMICS

UDC 330.45:658.7.01]:656.033
DOI https://doi.org/10.26661/2414-0287-2024-3-63-04

MATHEMATICAL MODEL OF THE PRODUCT SUPPLY PROCESS WITH UNCERTAIN DELIVERY PRICES

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Key words:
logistics systems, transportation logistics, key agent mechanism, mathematical model.

The article addresses the issue of product delivery under the condition that delivery costs are estimated based on the reports of economic agents. The cost of delivering goods along a specific route may be uncertain and dependent on the individual preferences of carriers. For instance, a carrier might prefer a route that passes by a store they want to visit, a dining facility, or one that is perceived as safer from the carrier's perspective. Naturally, only the carrier can determine this internal cost of transporting goods. The question arises: how can a system of delivery routes be chosen that serves as a compromise between the carriers' preferences? Formally, the task is to minimize the collective transportation costs as much as possible. Here, the costs are calculated based on the prices reported by individual economic agents, the carriers. However, the carriers' reports may not necessarily be truthful. To secure a more favorable route, a carrier might provide false information about transportation costs. How can agents be incentivized to provide truthful information? One approach to solving this problem is the key agent mechanism, which is discussed in this paper. As a result of the study, a mathematical model of the product supply process with uncertain delivery prices has been developed, which can be used in the development of decision support systems in transportation logistics.

МАТЕМАТИЧНА МОДЕЛЬ ПРОЦЕСУ ПОСТАЧАННЯ ПРОДУКЦІЇ З НЕВИЗНАЧЕНИМИ ЦІНАМИ ДОСТАВКИ

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Ключові слова:
логістичні системи, транспортна логістика, транспортна задача, механізм ключових агентів.

Стаття присвячена проблемі доставки продукції при умові, що вартість доставки оцінюється на базі повідомлень економічних агентів.
Вартість доставки продукції за конкретним маршрутом може бути невизначеною і базуватися на індивідуальних уподобаннях перевізників. Наприклад, перевізник може прагнути вибрати маршрут, який проходить повз магазин, який він хотів відвідати або повз їдальні, або просто безпечніший з точки зору конкретного перевізника. Звісно, цю внутрішню ціну перевезення товару з погляду окремого перевізника може визначити лише сам перевізник. Виникає питання: як вибрати таку систему маршрутів доставки в транспортній задачі, яка була б своєрідним компромісом між побажаннями перевізників. Формально завдання полягає в тому, щоб якомога зменшити колективні витрати на перевезення товару. Тут витрати розраховуються виходячи з цін, які повідомляють окремі економічні агенти – перевізники. Проте повідомлення перевізників не обов'язково будуть правдивими. Для того, щоб забезпечити собі більш вигідний маршрут, перевізник може передати хибне повідомлення про ціну перевезення. Як змусити агентів повідомляти правдиву інформацію? Одним із способів вирішення такого завдання є механізм ключових агентів, який і розглядатиметься в даній роботі.

ISSN Print 2786-5150
ISSN Online 2786-5169

Problem Statement

Let's consider the problem in the following formulation. There are nnn storage points where a homogeneous product is stored and mmm consumption points where this product is needed. Each storage point has a dedicated carrier responsible for transporting the product from that specific point. The product can be delivered from any storage point to any consumption point via a corresponding route. Each of the mnmnmn routes connecting the storage and consumption points has its own transportation cost for the respective carriers. The task is to find such a delivery plan that minimizes the total transportation costs, calculated based on the carriers' reported prices. However, the carriers' reports about the costs might be inaccurate.

Certain conditions must be considered mandatory:

1. The selection of the optimal system of routes is based on the agents' reports about transportation costs, reducing the problem to solving a transportation problem. This rule is known to all agents.
2. The behavior of the agents–the carriers–is non-cooperative, meaning the carriers cannot coordinate their reports, and no agent has information about the reports of other agents.
3. Agents may be subject to penalties, and the penalty amount is determined based on all agents' reports. The rule for determining penalties is known to all agents.
4. Each agent aims to minimize their individual costs, including penalties.

Review of Recent Research and Publications

The transportation problem discussed in this article belongs to the broader class of transportation logistics problems. Significant contributions to the development of transportation logistics management and the system for evaluating the efficiency of logistics activities have been made by Ukrainian scholars such as B.A. Anikin, V.G. Banko, A.V. Lozovyi, L.B. Myrotina, O.A. Novikov, M.A. Perebyinis, B.V. Shabov, and others [1-5].

The active development of logistics theory began in the first half of the 20th century. The modern perspective on transportation logistics started to take shape in the United States in the second half of the 20th century [2,4]. By the mid-1950s, the term "logistics" had become part of economic terminology. One of the main and integral components of the logistics system is transportation. Transportation is a crucial element of various economic processes (such as production, trade, etc.). Therefore, problems related to transportation management play a significant role and occupy a prominent place in logistics theory [6-13].

A number of factors define transportation logistics as an independent branch of economics:

1. The presence of transportation in all tasks related to managing product flows.
2. The inclusion of transportation logistics within the system: "supply – production – distribution – consumption."
3. The numerous transportation challenges associated with selecting distribution channels for raw materials, semi-finished products, and finished goods within the logistics system.
4. The large number of economic agents (transportation and forwarding companies) that play a significant role in organizing optimal delivery of goods, both in domestic transportation and international shipping.
5. The high share of transportation costs, which can reach 50% or more of the total logistics costs associated with moving goods from the primary source of raw materials to the end consumer of finished products.
6. The substantial share of the transportation component in the external trade price of goods, especially for countries with long transportation distances.

Transportation logistics enables the solution of three main tasks within this system [5], specifically tasks related to:

1. Forming service market zones, forecasting material flow, processing material flow within the serviced system (supplier's warehouse, consumer, wholesale trading company), and other activities related to the operational management and regulation of material flow.
2. Developing a system for organizing the transportation process (transportation plan, activity distribution plan, cargo flow formation plan, transportation schedule, etc.).
3. Managing inventories and servicing them with transportation means and information systems.
4. Mathematical methods in modern transportation logistics began to develop at the start of the 20th century [13-15]. Today, it is impossible to imagine solving transportation management problems without using mathematical models, methods, and information systems. The transportation models are examined in the works of scholars such as O.Yu. Zaichenko, Yu.P. Zaichenko, O.M. Isakova, O.M. Shevchenko, S.I. Nakonechny, and S.S. Savin.
5. One of the challenges in the practical application of scientific achievements in this field is the behavior of economic agents. The issue lies in the fact that agents do not necessarily convey accurate information in their reports. Each participant in the economic process seeks to improve their own utility. Rational behavior of an economic agent may involve providing false information for collective decision-making if it results in the enhancement of their own utility function. Consequently, a particular type of uncertainty arises in such problems, not due to the inaccuracy of initial data, but due to the deliberate misinformation provided by agents, participants in economic processes. This situation resembles a cooperative game with many participants [16,17]. One method to reduce this uncertainty is the key agent mechanism [18].

Objectives

The goal of this article is to develop a mathematical model for a transportation problem where transportation costs are determined based on the reports of carriers, which may contain inaccurate information. The task is to use the key agent mechanism to establish penalties for carriers that would discourage them from providing false information about transportation costs. In other words, in the absence of collusion among carriers, the total costs (including the penalty) for any individual carrier would be minimized only when truthful information is provided. This mechanism can be applied in organizing transportation tenders,

ISSN Print 2786-5150
ISSN Online 2786-5169

forecasting production costs, and other situations involving competition among carriers.

Presentation of the Main Research Material

Let us consider the classical formulation of the transportation problem involving *n* storage points and *m* consumption points for a product. We will assume that all possible delivery routes are permissible. Additionally, we will consider the scenario where information about transportation costs on these routes is provided by carrier agents.

We will assume the transportation problem is closed, meaning that the total supply at the storage points equals the total demand at the consumption points (otherwise, the transportation problem is open, which can easily be converted to a closed type). The criterion for evaluating the quality of the solution will be the total transportation costs.

Let the goods be stored in volumes $a_i \quad i=1,2,...,n$ at the storage points. At consumption points *j*, the volume of consumption is, respectively, $b_j \quad i=1,2,...,m$. The closedness condition of the transport problem has the form $\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_i$.

An acceptable solution to the problem is determined by the matrix $x=(x_{ij})_{i=1,2,...,n;\, j=1,2,...,m}$, where each element x_{ij} of the matrix is equal to the volume of transportation of goods from point *i* to point *j*. Taking into account the closure condition, we obtain the following standard constraints of the transport problem:

$$\sum_{j=1}^{m} x_{ij} = a_i \quad i=1,2,...,n\,,$$
$$\sum_{i=1}^{n} x_{ij} = b_j \quad j=1,2,...,m\,, \qquad (1)$$
$$x_{ij} \ge 0 \quad \forall i,j\,.$$

The criterion of the problem, taking into account the above assumptions, has the form:

$$F(x)=\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} \to \min\,. \qquad (2)$$

Prices $c_{ij} \quad j=1,2,...,m$ are "correct" prices related to the *i*-th carrier agent, which reflect the price of transporting a unit of goods to the *j*-th consumer. Moreover, each agent has his own cost criterion:

$$F_i(x)=\sum_{j=1}^{m} c_{ij}x_{ij} \to \min, \quad i=1,2,...,n. \qquad (3)$$

In order to obtain more profitable transportation routes, the *i*-th agent may transmit incorrect price messages. We will mark such messages $f_{ij} \quad j=1,2,...,m$. The non-cooperative behavior of agents means that when forming their messages, each agent assumes that the other agents will convey the true information.

Now we will attribute a fine to each of the agents, which will be calculated on the basis of the mechanism of key agents.

Let agent i_0 report information about its prices $f_{i_0 j} \quad j=1,2,...,m$, which may differ from real prices $c_{i_0 j} \quad j=1,2,...,m$. Assume that all other agents report the correct information.

Then the criterion in the optimization problem takes the following form:

$$F'(x)=\sum_{i\ne i_0}\sum_{j=1}^{m} c_{ij}x_{ij} + \sum_{j=1}^{m} f_{i_0 j}x_{ij} \to \min\,. \qquad (4)$$

Let's define the concept of a key agent in the considered problem.

We will call an agent *i* key, the removal of which (and, accordingly, the removal of the *i*-th item of goods storage) leads to the fact that the optimal solution of the transport problem (no longer closed) for other agents will change in such a way that their total costs will decrease compared to by those they had during the optimal solution of the primary problem.

We define the cost function of a coalition of agents without an agent with the number i_0:

$$F_{\bar{i}_0}(x)=\sum_{i\ne i_0}\sum_{j=1}^{m} c_{ij}x_{ij}\,. \qquad (5)$$

Consider the auxiliary transport problem with constraints (1) and objective function (5).

Let $x^*=(x_{ij}^*) \; i=1,2,...,n; j=1,2,...,m$ – the optimal solution of problem (1), (2), $x^{**}=(x_{ij}^{**}) \; i=1,2,...,n; j=1,2,...,m$ – the optimal solution of problem (1), (4), obtained when the agent with number i_0 incorrectly reports information, and $x^{***}=(x_{ij}^{***}) \; i=1,2,...,n; j=1,2,...,m$ – the optimal solution of problem (1), (5) Let us determine the amount of the fine for agent with number i_0 as follows:

$$S_{i_0}=F_{\bar{i}_0}(x^{**})-F_{\bar{i}_0}(x^{***}) \qquad (6)$$

We note that from definition (6) the inequality $S_{i_0} \ge 0$ holds. The agent is key if and only if. $S_{i_0} > 0$ We also note that the amount of the fine is determined only taking into account agents' reports (in particular, false ones).

Consider the total costs of the agent with number i_0.

$$F_{i_0}(x^{**})+S_{i_0}=F_{i_0}(x^{**})+F_{\bar{i}_0}(x^{**})-F_{\bar{i}_0}(x^{***})=$$
$$=\sum_{j=1}^{m} c_{i_0 j}x_{i_0 j}^{**}+\sum_{i\ne i_0}\sum_{j=1}^{m} c_{ij}x_{ij}^{**} - F_{\bar{i}_0}(x^{***})=$$
$$=\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij}^{**}-\sum_{i\ne i_0}\sum_{j=1}^{m} c_{ij}x_{ij}^{***}=F(x^{**})-F_{\bar{i}_0}(x^{***})\ge$$
$$\ge F(x^*)-F_{\bar{i}_0}(x^{***})\ge F(x^*)-F_{\bar{i}_0}(x^*)$$

From this inequality and definition (2), (5) of the functions $F(x)$ and $F_{\bar{i}_0}(x)$, the following inequality follows:

$$F_{i_0}(x^{**})+S_{i_0}\ge F_{\bar{i}_0}(x)=\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij}^{*}-$$
$$-\sum_{i\ne i_0}\sum_{j=1}^{m} c_{ij}x_{ij}^{*}=\sum_{j=1}^{m} c_{i_0 j}x_{i_0 j}=F_{i_0}(x^*)$$

Therefore, the expenses of the agent with number i_0, provided that he is informed of incorrect information, is not less than his expenses when he is informed of correct

ISSN Print 2786-5150
ISSN Online 2786-5169

information. Thus, with non-cooperative behavior, it makes no sense for each of the agents to report incorrect information (it does not reduce his costs).

Conclusions

This paper presents a mathematical model for freight delivery under conditions where transportation costs are determined by the carrier agents themselves. Under the assumption of non-cooperative behavior among agents, the study determines the penalty size at which it is not advantageous for any agent to provide false information about their prices. The optimal solution to the problem and the penalties are based on the agents' reports. The key agent mechanism forms the basis for determining these penalties.

The mathematical model can be generalized to more complex cases of distributed delivery under uncertain pricing conditions. The developed model of the product supply process with uncertain delivery prices can be utilized in the development of decision support systems in transportation logistics.

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