

UDC 539.3

## DEFORMATION AND STABILITY OF THIN-WALLED SHALLOW CONICAL SHELLS UNDER EXTERNAL PRESSURE AND PERIODICALLY DISCRETE BORDER FIXATION

Bessmertnyi Y. O., post-graduate student

*Prydniprov's'ka State Academy of Civil Engineering and Architecture,  
Chernychevskiy str. 24a, Dnipro, 49600, Ukraine*

yaroslavbessmertnyi@gmail.com

A numerical study (within ANSYS software) deals with deformation and buckling of series of elastic isotropic flat closed conical shells subject to external normal uniform pressure. Fixation of shells edges is periodically discrete (movable-support sections alternate with hinged-support sections of the same length). Length of each periodic fixation is the same, and the total length of mobile fixation is equal to the total length of immobile fixation. While loading, the type of shells fixation causes periodically non-uniform stress-strain state in the circumferential direction. Periodicity of subcritical stress-strain state is equal to a number of fixed sections that varies in a wide range from zero up to fourteen and infinite. In this case zero corresponds to continuous mobile fixation, infinite – continuous immobile fixation, from two up to fourteen corresponds to periodic boundary conditions. The phenomenon of “static resonance” was found for conical shells. The essence of the effect is as follows. Minimum limit pressure of geometrically nonlinear analysis corresponds to the periodicity of stress-strain state of a conical shell which coincides with a half-sum of the periodicity of the first eigenvibration mode of an unloaded shell and the first eigenmode of linear buckling problem of a continuously hinged supported shell subject to external pressure. All results are presented in form of graphs and tables for each type of calculations – linear problem of stability (bifurcation) and geometrically non-linear problem of stability. The results of this work make possible to forecast the “static resonance” phenomena for shell with heterogeneous stress-strain state on the basis of two simple linear calculations.

*Key words: thin-walled shallow conical shell, periodically discrete fixation, heterogeneous stress-strain state, software ANSYS.*

## ДЕФОРМУВАННЯ ТА СТІЙКІСТЬ ПОЛОГИХ КОНІЧНИХ ТОНКОСТІННИХ ОБОЛОНОК ПРИ ЗОВНІШНЬОМУ ТИСКУ ТА ПЕРІОДИЧНО ДИСКРЕТНОМУ СПИРАННІ

Бессмертный Я. О., аспирант

*Придніпровська державна академія будівництва та архітектури,  
вул. Чернишевського, 24а, м. Дніпро, 49600, Україна*

yaroslavbessmertnyi@gmail.com

Чисельне дослідження (у програмному комплексі ANSYS) присвячено деформації і стійкості ряду пружних ізотропних плоских замкнутих конічних оболонок під дією зовнішнього тиску. Фіксація краю оболонок періодично дискретна (секції рухомої опори чергуються з секціями шарнірної нерухомої опори однакової довжини). При навантаженні тип кріплення оболонок викликає періодично нерівномірний напружено-деформований стан в окружному напрямку. Періодичність докритичного напружено-деформованого стану дорівнює числу фіксованих ділянок, які варіюються в широких межах. Було виявлено явище «статичного резонансу» для конічних оболонок. Ефект полягає в такому: мінімальний граничний тиск геометрично нелінійного аналізу відповідає періодичності напружено-деформованого стану конічної оболонки, яка збігається з напівсумою періодичності першої моди власних коливань ненавантаженої оболонки і першої власної моди лінійної задачі стійкості оболонки з однорідним шарнірно нерухомим закріпленням при зовнішньому тиску.

*Ключові слова: полого тонкостінна оболонка, періодично дискретне закріплення, неоднорідний напружено-деформований стан, програмний комплекс ANSYS.*

## ДЕФОРМИРОВАНИЕ И УСТОЙЧИВОСТЬ ПОЛОГИХ КОНИЧЕСКИХ ТОНКОСТЕННЫХ ОБОЛОЧЕК ПРИ ВНЕШНЕМ ДАВЛЕНИИ И ПЕРИОДИЧЕСКИ ДИСКРЕТНОМ ОПИРАНИИ

Бессмертный Я. О., аспирант

*Приднепровская государственная академия строительства и архитектуры,  
ул. Чернышевского, 24а, г. Днепр, 49600, Украина*

yaroslavbessmertnyi@gmail.com

Численное исследование (в программном комплексе ANSYS) посвящено деформации и устойчивости ряда упругих изотропных плоских замкнутых конических оболочек, подверженных внешнему давлению. Фиксация края оболочек периодически дискретна (секции подвижной опоры чередуются с секциями шарнирной неподвижной опоры одинаковой длины). При нагружении тип крепления оболочек вызывает периодически неравномерное напряженно-деформированное состояние в окружном направлении. Периодичность докритического напряженно-деформированного состояния равна числу фиксированных участков, которые варьируются в широких пределах. Было обнаружено явление «статического резонанса» для конических оболочек. Суть эффекта заключается в следующем: минимальное предельное давление геометрически нелинейного анализа соответствует периодичности напряженно-деформированного состояния конической оболочки, которая совпадает с полусуммой периодичности первой моды собственных колебаний незагруженной оболочки и первой собственной моды линейной задачи устойчивости оболочки с однородным шарнирно неподвижным закреплением при внешнем давлении.

*Ключевые слова: пологая тонкостенная коническая оболочка, периодически дискретное закрепление, неоднородное напряженно-деформированное состояние, программный комплекс ANSYS.*

### INTRODUCTION

In [1-3] based on experimental studies of deformation and stability of axially compressed elastic isotropic circular cylindrical shells in the presence of a periodically non-uniform stress-strain state (SSS) in the circumferential direction, the phenomenon called “static resonance” was discovered. Non-uniform SSS of smooth shells was caused by periodic longitudinal compressive forces or periodic middle shell surface imperfections (for uniform axial compression or external pressure) [1-3]. Precritical uniform SSS of longitudinally reinforced shell subject to axial compression was caused by versatile arrangement of reinforcements (stringers).

The essence of “static resonance” consisted in the following. The maximal precritical displacements and minimum limit load was realised for static compression of shells when periodicity of non-uniform SSS in the circumferential direction coincided or was closed to periodicity of first eigenvibration mode of a shell without any load. Numerical calculations [4] using ANSYS software also confirmed the presence of “static resonance”.

The analogue of “static resonance” was discovered in the numerical study [5] (ANSYS software) for shallow conical shells under external pressure. Non-uniform SSS of the shells was caused by large (more than 10 thicknesses) periodic mid-surface imperfections in the circumferential direction. Unlike the earlier problems, “static resonance” of conical shells did not appear for the limit pressure. But it revealed itself in the linear buckling problem. As before, this “resonance” was realised when a periodicity of non-uniform SSS coincided with the periodicity of the first eigenvibration mode of a shell.

Note that the calculation improvement of bearing capacity of the shells under external pressure in case of non-uniform SSS is an important and actual task [6-7]. This is especially important for shallow shells which are widely used in practice.

The aim of this work is to study (using ANSYS software) the influence of periodically discrete border fixation of shallow conical shells under uniform external pressure on deformation and buckling, as well as effects that are caused by non-uniform SSS.

We performed three types of analysis: 1) linear bifurcation analysis to determine the critical pressure value  $q^{cr}$  and corresponding eigenmodes; 2) dynamic analysis to determine first transversal eigenvibration modes of a shell without any load; 3) geometrically nonlinear analysis to determine SSS of a shell, its limit pressure value  $q^{lim}$  and corresponding buckling modes. All analyses were executed both for shells with periodically discrete border fixation (movable-support sections alternate with hinged-support sections of the same length) and for shells with continuous movable support or hinged support. The number of movable and hinged sections ( $n$ ) of the shell edge varied from 0 to 14 in increments of 1.

## MAIN PART

### Methodology of numerical analysis

As already mentioned, the numerical study was carried out within ANSYS software (ANSYS Inc. Academic Research, Mechanical Analysis, Release 13.0 customer 298728). Thin-walled closed flat cones (Fig. 1) had next parameters: shells thickness  $h = 5$  mm; base radius  $R = 2000$  mm; height  $H = 140$  mm. Angle between conical shell generatrix and its base is  $\alpha = 4^\circ$ .

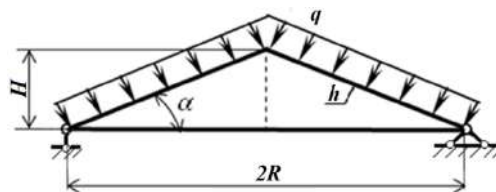


Fig. 1. Shell scheme with movable and hinged support

Material of shells is steel X18H9H (former Soviet Union steel specification) with following mechanical characteristics: modulus of elasticity  $E = 2 \times 10^5$  MPa, Poisson's ratio  $\nu = 0.3$ , yield stress  $\sigma_{02} = 800$  MPa, material density  $\rho = 7850$  kg/m<sup>3</sup> (the density is taken into account in the analysis 2).

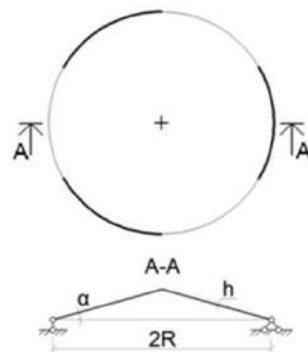


Fig. 2. Schema of conical shell with periodically discrete edge fixation  $n = 3$  (bold line – hinged support, fine line – movable support)

Modelling of the problems was conducted in the following order: 1) spatial geometric model of shells was created by rotation of the generatrix about the axis of symmetry ( $Y$ -axis); 2) models were divided into identical sectors, which number was assumed to be  $2n$  ( $n$  varied from 0 to 14 in increments of 1); 3) the boundary conditions corresponded to alternately movable or hinged support and were set at the edge of the curved portion of each sector; 4) external uniform normal pressure  $q$  was applied; 5) finite element mesh was created using shell finite element (FE) SHELL 281 (from ANSYS standard library). It should be emphasised that hinged support restricted displacements of the edges along three axes  $X$ ,  $Y$ ,  $Z$ , and movable support of edges confined only vertical displacements along  $Y$ -axis.

FE meshing of the shells was performed by an internal function of ANSYS software that divides the shell surface into finite elements. The main parameters affecting the creation of the FE model were set as: FE size, FE mesh type (mapped or adaptive), FE shape (triangular or rectangular). The

density of a mesh affects the magnitude of shell surface displacements and values of critical  $q^{cr}$  and limit  $q^{lim}$  pressures. FE mesh was refined until pressure values increment became stable, less than 0.5%. As a result, meshed models consisted of FE  $N = 1400..3680$  (depending on the value of  $n$ ).

### RESULTS OF NUMERICAL ANALYSIS

Critical pressure  $q^{cr}$  and corresponding eigenmodes ( $m$  – a number of waves in the circumferential direction), first eigenvibration modes ( $k$  – a number of waves in circumferential direction) were obtained within analyses 1 and 2 for three types of shells fixation: with periodically discrete edge fixation ( $n = 1 \dots 14$ ), with continuous movable support ( $n = 0$ ) and with continuous hinged support ( $n = \infty$ ). The values of  $q^{cr}$ ,  $m$  and  $k$  for the shells are placed in Table 1.

Table 1 – Results of numerical analyses for shell series ( $R/h = 400$ ,  $\alpha = 4^\circ$ ,  $h = 5$  mm)

$n$	0	1	2	3	<u>4</u>	5	6	7	8	9	10	11	12	13	14	$\infty$
$q^{cr}$ , kPa	2.58	2.13	2.87	2.99	3.05	3.12	3.23	3.26	3.23	3.20	3.19	3.33	3.41	3.44	3.46	3.59
$m$	4	2	4	4	4	5	5	5	4	4	5	5	5	5	5	<u>5</u>
$\omega$ , Hz	16.1	16.5	18.3	19.7	21.2	21.6	21.6	22.2	22.7	22.8	22.9	22.9	23.0	23.1	23.2	23.4
$k$	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	<u>3</u>
$q^{lim}$ , kPa	1.71	2.01	3.53	2.56	<u>2.28</u>	2.75	3.28	3.77	4.23	4.61	4.91	5.16	5.36	5.51	5.62	5.93

The solution of geometrically nonlinear problem was to build dependences of an external pressure on the transverse displacements of a shell surface (first of all, displacement  $w_z$  of the shell top). When pressure value reaches  $q^{lim}$  in case of constant pressure, a shell realises an abrupt transition to a noncontiguous equilibrium mode (reverse to its initial position) that belongs to an up-going stable post-critical branch of equilibrium state. Values of limit pressure  $q^{lim}$  for all types of shell fixation including shells with  $n = 0$  and  $n = \infty$  are shown in Table 1.

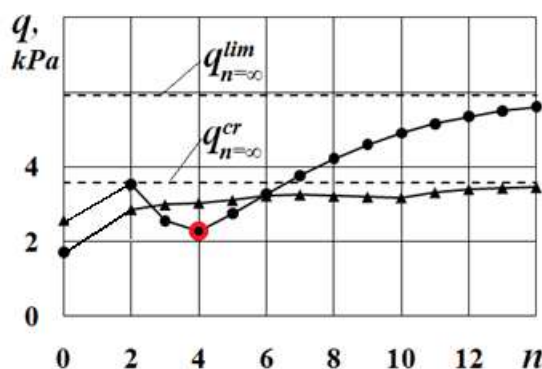


Fig. 3. Dependencies “ $q^{cr}, q^{lim} - n$ ” for conical shells

Visualisation of SSS at various stages of loading shows that for conical shells periodical SSS in the circumferential direction caused by discrete fixation involves almost the entire shell surface.

Using table data for conical and spherical shells, the dependences of critical and limit pressures on number of periodically discrete fixations “ $q^{cr}, q^{lim} - n$ ” has been plotted (Fig. 3). On the graphs, triangles represent critical pressure  $q^{cr}$  and circles refer to limit pressure  $q^{lim}$ . We have connected the values of pressure with straight lines for a clear perception. The values of critical ( $q_{n=\infty}^{cr}$ ) and limit ( $q_{n=\infty}^{lim}$ ) pressure of a shell with continuous hinged support are represented by horizontal dashed lines.

Table 1 and dependences in Figure 3 show that for conical shells limit pressures  $q^{\text{lim}}$  lie in the range of  $(q_{n=0}^{\text{lim}} - q_{n=\infty}^{\text{lim}})$  and are much higher than critical pressures  $q^{\text{cr}}$  ( $q_{n=0}^{\text{cr}} - q_{n=\infty}^{\text{cr}}$ ). Moreover, the range of values  $q^{\text{lim}}$  includes the range of values  $q^{\text{cr}}$ . Thus, limit pressure  $q^{\text{lim}}$  is minimum buckling pressure of a conical shell with continuous hinged support. The dependence “ $q^{\text{cr}} - n$ ” is mostly up-going except some fluctuations of distinct points. Increase of critical pressure is comparatively small and corresponds to  $q^{\text{cr}} = 2.58 - 3.59$  kPa. Values of  $q^{\text{cr}}$  for maximum number of discrete hinged supports ( $n = 14$ ) are less than the values for continuous hinged support. The difference is about 3.6%.

The dependence “ $q^{\text{lim}} - n$ ” obtained as a result of geometrically non-linear analysis is more complicated comparing to linear analysis. First of all, this dependence reaches its minimum for continuous movable support ( $q_{n=0}^{\text{lim}}$ ). In addition, it has an intermediate minimum for  $q_{n=4}^{\text{lim}}$ . In this case limit pressure turns out to be lower than critical pressure. The presence of local minimums of limit pressure values changed the situation: limit pressure  $q^{\text{lim}}$  becomes dangerous. And the region where limit pressure is dangerous includes three neighbour sections  $n = 3 - 5$ . As we noted in Introduction, the above-described phenomenon was called “static resonance” in the papers [1-5]. It was discovered in physical and numerical experiments for circular cylindrical [1-4] and conical shells [5] with periodically non-uniform SSS in the circumferential direction. In the case of conical shells the minimum “resonance” value of limit pressure  $q_{\text{min}}^{\text{lim}}$  is the most dangerous for discrete edges fixation. Here, “static resonance” is realised for  $n = 4$  (bigger sign corresponds to the “resonance” in Fig. 3). The value of “resonance” pressure is 25% less than the value of critical pressure  $q^{\text{cr}}$ . The “resonance” values of  $n$  and limit pressure ( $q_{\text{min}}^{\text{lim}}$ ) are bold and underlined in Table 1.

Note that in [1-5] “static resonance” was only associated with a coincidence (or with a proximity) of periodicity of precritical SSS in the circumferential direction and periodicity  $k$  of first eigenvibration mode of an unloaded shell. But for conical shells “static resonance” appears definitely when the periodicity of precritical SSS in the circumferential direction is equal to a half-sum of the periodicity  $k$  of the first eigenvibration mode of an unloaded shell and periodicity  $m$  of the first eigenmode of linear buckling problem of a continuously hinged supported shell subject to external pressure. Values  $m$  and  $k$  for  $n = \infty$  are shown in bold italic in Table 1.

## CONCLUSIONS

The results of the present research let one predict “static resonance” of shallow conical shell subject to external pressure in the case of periodically non-uniform SSS using two simple linear analyses. In particular, “static resonance” appears when periodicity of SSS in the circumferential direction coincides with a half-sum of waves of the first eigenvibration mode of an unloaded shell and waves of the first eigenmode of linear buckling problem of a shell subject to external uniform pressure. In both cases, these calculations must be carried out for continuous hinged support of shell edges.

Note that few values  $q^{\text{lim}}$  are presented in the region of dangerous limit pressures, and they are also dangerous for shells along with minimum (resonance) value of limit pressure  $q_{\text{min}}^{\text{lim}}$ . Therefore, we recommend moving out this region as much as possible by increasing the periodicity of SSS away from “resonance” periodicity i.e. by increasing a number of hinged supports.

Design of shells with small number of periodically discrete edge fixation assumes a choice of analysis. It is necessary to focus on limit pressure obtained in geometrically non-linear analysis for small angles  $\alpha$  (for shallow shells). While for large angles (for deep shells) a dangerous load is critical pressure  $q^{\text{cr}}$  obtained in linear buckling analysis (bifurcation).

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УДК: 004.056.55

## ЗАХИСТ ПЕРСОНАЛЬНИХ ДАНИХ З ВИКОРИСТАННЯМ АЛГОРИТМІВ НЕОДНОЗНАЧНОГО ШИФРУВАННЯ

Гальченко А. В.

*Запорізький національний університет,  
вул. Жуковського, 66, м. Запоріжжя, 69600*

andrem1993@ukr.net

У статті розглядається проблема захисту персональних даних, яка є досить актуальною в сучасному світі. Ця проблема висвітлюється у засобах масової інформації різними державними службами, які займаються питаннями безпеки, і фахівцями у галузі інформаційної безпеки. Автором статті висвітлюється ідея використання засобів неоднозначного шифрування даних для захисту інформації в ІСПДн. У процесі дослідження автор розглянув проблеми захисту персональних даних, зробив аналіз літератури та ресурсів у мережі Інтернет, на яких описано традиційні і сучасні механізми їх захисту, а та також виконав огляд літератури, аналіз алгоритмів неоднозначного шифрування, обрав серед них найбільш адаптований для виконання поставленої задачі та порівняв його ефективність з традиційними і сучасними підходами до захисту інформації в ІСПДн. Для наведення практичних результатів автор виконує шифрування/дешифрування тестової ІСПДн з використанням традиційних, сучасних і неоднозначних засобів шифрування даних. Описані дослідження проводилися аспірантом кафедри програмної інженерії Запорізького національного університету в рамках дисертаційної роботи «Дослідження інструментальних механізмів заперечуваного шифрування».

*Ключові слова: персональні дані, інформаційна система, деперсоналізація, шифрування, неоднозначне шифрування, примушування, перемішування даних, bigdata.*