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FRACTURE ANALYSIS AND SHIELDING EFFECTS IN ADVANCED MATERIALS

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Due to their ability to interconnect electrical and mechanical domains, piezoelectric materials are increasingly finding applications in advanced electromechanical systems. Notably, magneto-electric composites like BaTiO3-CoFe2O4 have emerged as vital materials for the next generation of devices. Exploring this realm is a complex and significant endeavor, demanding the attention of the scientific community and further research. Over recent decades, the scope of formulating and executing computational models for continuous media has substantially broadened, encompassing a diverse array of material properties and fields in computational models.

This article centers on the utilization of asymptotic analysis as a mathematical tool for constructing approximate equations and assessing the relevance of various hypotheses. It delves into the utilization of the perturbation method,

pioneered by Kagadiy T. S. and others, to tackle two-dimensional contact problems in electropelasticity, particularly in the context of materials with linear anisotropy. The extensive applicability of this asymptotic approach underscores its efficacy in simplifying intricate problems by breaking them down into a sequence of boundary problem resolutions grounded in the theory of potentials.

The collaborative effort of the authors in this research underscores that employing the mentioned method opens up avenues for formulating pertinent boundary problems for the fundamental equations. This, in turn, allows for the representation of the initial electropelasticity problem as a superposition of more manageable boundary problems. While mechanical and electrical components can be treated separately, they still interact via boundary conditions.

To conduct a numerical analysis, the article examines scenarios where one crack surface slides parallel to the crack front against another and selects relevant materials with known characteristics. Calculations reveal that, even in the absence of mechanical load, crack surfaces undergo relative sliding due to non-zero electrical or magnetic fields. The study of the shielding effect, which mitigates crack motion by altering magnetic field distribution, holds particular significance. This effect reduces the overall stress intensity factor and impedes crack propagation.

Conducting such a comprehensive analysis is pivotal for comprehending and foretelling the strength and reliability of structural components composed of piezoelectric and piezomagnetic materials. It is imperative to conduct a thorough examination of the mechanisms governing their failure.

АНАЛІЗ ТРІЩИН ТА ЕФЕКТИ ЕКРАНУВАННЯ В СУЧАСНИХ МАТЕРІАЛАХ

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Ключові слова:
інтелектуальні матеріали, механіка руйнування, розрахункові моделі, ефект екранування, електропружність.

П'єзоелектричні матеріали, завдяки їхньому зв'язку між електричними та механічними полями, знаходять все більше використання в інтелектуальних електромеханічних системах. Магнітоелектричні композити, такі як BaTiO3-CoFe2O4, стали важливими матеріалами нового покоління пристроїв.
Пошуки у даному напрямку досить складні і важливі, вони вимагають уваги наукової спільноти та подальших досліджень. Протягом останніх кількох десятиліть можливості для формулювання і реалізації обчислювальних моделей для неперервних середовищ значно розширилися за рахунок охоплення великого спектра фізичних властивостей речовин та полів в обчислювальних моделях.
У цій статті головну роль відіграє асимптотичний аналіз як математичний інструмент для створення наближених рівнянь та оцінки значущості різних гіпотез. Розглядається застосування методу збурення, розробленого Кагадій Т. С. та іншими, для вирішення двовимірних контактних задач електропружності, зокрема для матеріалів із прямолінійною анізотропією. Широкий спектр застосувань даного асимптотичного підходу демонструє його ефективність для спрощення складних задач, зводячи їх до послідовного розв’язання крайових задач, заснованих на теорії потенціалу. Колективом авторів даного дослідження показано, що завдяки застосуванню вказаного методу відкривається можливість формулювання відповідних крайових задач для основних рівнянь і у подальшому представити початкову задачу електропружності у вигляді суперпозиції більш простих крайових задач. При цьому механічні та електричні складові можуть бути відокремлені, але зберігають взаємодію через крайові умови.
Для проведення чисельного аналізу авторами статті було розглянуто випадок з ковзанням однієї поверхні тріщини по іншій паралельно фронту тріщини та обрані актуальні матеріали з відомими характеристиками. З результатів розрахунків видно, що навіть за нульового механічного навантаження грані тріщини ковзають одна відносно одної за рахунок ненульових електричних або магнітних полів. Вельми цікавим для вивчення є ефект екранування, який пом'якшує рух тріщини шляхом зміни розподілу магнітного поля. Це зменшує загальний фактор інтенсивності напружень і запобігає поширенню тріщини.
Проведення подібного комплексного аналізу є вкрай важливим для розуміння та передбачення міцності та надійності структурних компонентів, виготовлених із п'єзоелектричних / п'єзомагнітних матеріалів, важливо провести комплексний аналіз механізмів їх руйнування.

Introducton. Over the past few decades, there has been a continuous and substantial expansion in the realm of computational models focusing on continuous media. These models have been consistently evolving to encompass an increasingly comprehensive array of physical attributes exhibited by substances and fields within them. A pivotal aspect of this expansion involves the development of techniques for the computation and design of intelligent materials with the capacity to autonomously adapt and optimize their properties. Notably, piezoelectric materials, which exhibit a fundamental connection between electric and mechanical fields, have found growing utility in intelligent electromechanical systems, serving as sensors, transducers, and actuators.

In the realm of advanced materials, magnetoelectric composites have gained significant prominence by combining piezoelectric and piezomagnetic components, like BaTiO3-CoFe2O4, through ceramic or nanotechnological methods. These composites exhibit the magnetoelectric effect resulting from the interaction between these phases, offering lightweight, strong, reliable, and environmentally resistant structural elements for next-generation intelligent devices. However, imperfections or deviations in their production or operation can lead to meso- and macroscopic defects, often resulting in composite failure. Anticipating the strength and dependability of structural components made from piezoelectric/piezomagnetic materials necessitates a thorough examination and analysis of their deterioration mechanisms, considering diverse crack models and conditions.

Recent research has focused on fractures initiating in piezomagnetic materials near the crack tip, particularly emphasizing the investigation of the shielding effect. This effect, reducing the overall stress intensity factor and inhibiting crack propagation, is driven by the magnetic field distribution around the crack tip [1]. The numerical integration considering this shielding effect encompasses the interplay between magnetic and mechanical fields, factoring in piezomagnetic material characteristics, crack geometry, and applied loads. Additionally, in addressing the limited relevance of electroelasticity to isotropic materials, there's a natural shift towards anisotropic materials, especially active ones like piezoelectric and piezoelectric electromagnetic materials [2]. These materials play vital roles in electronic devices, given their ability to change shape under electric or magnetic fields, despite their small dimensions and exposure to significant mechanical, electric, and magnetic forces. Furthermore, the advancement in nanotechnologies, crucial for the development of intricate magnetoelectric composites, has propelled the exploration of magnetoelectric elastic materials. The incorporation of the crack tip shielding effect amplifies the significance of this research, presenting a complex challenge that demands extensive further exploration [3].

In this study, the effectiveness of asymptotic analysis as a mathematical tool takes center stage, enabling researchers to construct approximation equations and assess the relevance of various hypotheses. Building upon the pioneering perturbation method developed by Shporta A. H. *et al.* [4], the research proposes its application to tackle two-dimensional contact problems of electroelasticity. Specifically, the study focuses on electroelastic materials characterized by rectilinear anisotropy. The broad applicability of this asymptotic approach showcases its potential to streamline complex problems, demonstrating the reduction of such challenges to a sequential solution of boundary value problems rooted in potential theory.

Statement of the problem. Let two mutually perpendicular planes of elastic symmetry pass through each point of a homogeneous anisotropic plate. Assuming that these planes are perpendicular to the Cartesian coordinate axes, x, y we obtain the following equations of equilibrium, electrostatics, electroelastic state, and Cauchy relations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0\,; \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0\,; \qquad (1)$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0\,; \quad \frac{\partial Э_x}{\partial y} - \frac{\partial Э_y}{\partial x} = 0; \qquad (2)$$

$$e_x = s_{11}^{D}\sigma_x + s_{12}^{D}\sigma_y + g_{11}^{\sigma_0} D_x\,;$$

$$e_y = s_{21}^{D}\sigma_x + s_{22}^{D}\sigma_y + g_{12}^{\sigma_0} D_x\,;$$

$$\gamma_{xy} = s_{66}^{D}\tau_{xy} + g_{26}^{\sigma,D} D_y\,;$$

$$E_x = -g_{11}^{\sigma,D}\sigma_x - g_{12}^{\sigma,D}\sigma_y + \beta_{11}^{\sigma} D_x\,; \; E_y = -g_{26}^{\sigma,D}\tau_{xy} + \beta_{22}^{\sigma} D_y\,; \qquad (3)$$

$$\frac{\partial}{\partial}\,; \qquad \frac{\partial}{\partial}\,; \; \gamma_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}. \qquad (4)$$

Here, σ_x, $\sigma_y\left(\tau_{xy}\right)$ – normal (tangential) stresses; U, V – components of the plate displacement vector; D_x, D_y i and $Э_x$, $Э_y$ – components of the induction vector and electric field intensity; $g_{kj}^{\sigma,D}$ – piezoelectric modulus of deformation and tension, measured at constant induction voltages; β_{kl}^{σ} – coefficients of dielectric susceptibility measured at constant stresses; s_{ij}^{D} – coefficients of deformation of the material of the body, measured at constant induction of the electric field.

It follows from the first equation of system (2) that there is some scalar function $\phi = \phi(x, y)$ such that

$$D_1 = D_x = \frac{\partial \phi}{\partial y}\,;$$

$$D_2 = D_y = -\frac{\partial \phi}{\partial x}\,.$$

The solution of one or another boundary value problem can be reduced to the integration of a system of equations under appropriate boundary conditions.

$$\begin{gathered} U_{xx} + \varepsilon U_{yy} + \varepsilon m V_{xy} - \left(a_{11} - \varepsilon a_{26}\right)\phi_{xy} = 0; \\ \varepsilon V_{xx} + q V_{yy} + \varepsilon m U_{xy} + \varepsilon a_{26}\phi_{xx} - q a_{12}\phi_{yy} = 0; \\ -\left(a_{11} - \varepsilon a_{26}\right)U_{xy} + \varepsilon a_{26}V_{xx} - q a_{12}V_{yy} + \\ + \varepsilon b_{22}\phi_{xx} + b_{11}\phi_{yy} = 0; \end{gathered} \qquad (5)$$

$$\varepsilon = \frac{G}{B_1}\,; \qquad \text{—}\,; \; m = 1 + \frac{\nu_2 B_1}{G} = 1 + \frac{\nu_1 B_2}{G}\,;$$

$$a_{11} = g_{11}^{\sigma,D} + v_2 g_{12}^{\sigma,D}\,;$$

$$a_{12} = g_{12}^{\sigma,D} + v_1 g_{11}^{\sigma,D}\,; \; a_{12} = g_{12}^{\sigma,D} + v_1 g_{11}^{\sigma,D}\,;$$

$$b_{22} = a_{26}^{2} + \beta_{22}^{\sigma}\frac{\sigma}{G}\,;$$

$$b_{11} = g_{11}^{\sigma,D} a_{11} + g_{12}^{\sigma,D} a_{12}\frac{B_2}{B_1} + \beta_{11}^{\sigma}\frac{\sigma}{B_1}\,,$$

The components of the stress tensor and the stress vector in this case are written as follows:

$$\sigma_1 = B_1\left(U_x + v_2 V_y - a_{11}\phi_y\right);$$

$$\sigma_2 = B_2\left(v_1 U_x + V_y - a_{12}\phi_y\right);$$

$$\tau = G\left(U_y + V_x + a_{26}\phi_x\right);$$

$$E_1 = -B_1 a_{11} U_x - B_2 a_{12} V_y + B_1 b_{11}\phi_y; \qquad (6)$$

$$E_2 = -G a_{26} U_y - G a_{26} V_x + G b_{22}\phi_x.$$

Here $\sigma_1 = \delta\sigma_x$, $\sigma_2 = \delta\sigma_y$, $Э_1 = \delta Э_x$, $Э = \delta Э$, $Э_2 = \delta Э_y$, $B_1 = \frac{E_1\delta}{1 - v_1 v_2}$, $B_2 = \frac{E_2\delta}{1 - v_1 v_2}$, $G = G_*\delta$, δ – is the thickness of the plate. The indices in equations (5) and ratios (6) denote differentiation by coordinates; E_1, E_2 – modulus of elasticity along the main directions x, y; G_x – shear modulus; v_1, v_2 – Poisson's ratios.

The method of solving the given problem. In real orthotropic materials, the value is $\varepsilon = \frac{G}{B_1}$ always much smaller than unity. The value $q = \frac{B_2}{B_1}$ can be considered as a small parameter during the asymptotic integration of the system (5). This assumption can be made because the ratio $q = \frac{B_2}{B_1}$ can be different ($q \le 1$ or $q \ge 1$), but always remains greater than ε.

Therefore, the value q will be concidered as of the order of one.

To take into account all possible differences between the values of the sought functions and their rates of change along the coordinates in elastic materials, x, y affine transformations of coordinates and sought functions are introduced

$$\xi_1 = \alpha\varepsilon^{1/2}x;\ \eta_1 = y;\ U = U^{(1)};$$

$$V = \varepsilon^{3/2}V^{(1)};\ \phi = \varepsilon^{3/2}\phi^{(1)}; \tag{7}$$

$$\xi_2 = x;\ \eta_2 = \beta\varepsilon^{1/2}y;\ U = \varepsilon^{3/2}U^{(2)};$$

$$V = V^{(2)};\ \phi = \varepsilon^2\phi^{(2)}. \tag{8}$$

By observing transformations (7) and (8), it becomes apparent that the solutions of the system of equations derived from (5) and modified using these transformations (either 7 or 8) exhibit a relatively gradual change along the respective coordinate, in contrast to similar solutions obtained by employing different transformations.

The overall tangential stress is the combination of both components. It is this stress that facilitates the connection between these two types of stress states. Depending on the applied load, one of them exhibits characteristics resembling a boundary layer.

Therefore, when subjecting piezomaterials to mechanical loads and specifying boundary conditions in terms of stresses, displacements, or their combinations, the solutions to the respective boundary value problems will manifest as a combination of solutions for these two stress-strain states:

$$U = U^{(1)} + U^{(2)};\ V = V^{(1)} + V^{(2)};\ \phi = \phi^{(1)} + \phi^{(2)}.$$

It is necessary to select appropriate asymptotic sequences to look for functions,, in the form of a power series of the parameter. The type of asymptotic sequence is determined by the structure of equations (5) and the order of the error ε in the boundary conditions that occurs after solving the problem in the zero approximation $(\varepsilon \to 0)$. To take into account all possible cases, we will define these functions in the form of series by parameter $\varepsilon^{1/2}$ (from transformations (7), (8) it is clear that series by lower powers of the parameter α cannot occur)

We will also present the coefficients α and β in the form of rows by parameter $\varepsilon^{1/2}$. After splitting the obtained system by the $\varepsilon^{1/2}$ parameter we arrive at an infinite system of equations concerning the basic functions $U^{1,j}$, $V^{1,j}$, $\phi^{1,j}$ $(j = 0,1,...)$. The auxiliary functions $V^{1,j}$, $U^{2,j}$, $\phi^{1,j}$, $\phi^{2,j}$ through the main ones are expressed by simple integration. At the same time, we will assume that $\alpha_{11} \sim \varepsilon\beta_{11}$, $b_{22} \sim \varepsilon^2 b_{11}$, $a_{12} \sim a_{26} \sim \varepsilon^3 b_{11}$. We present these equations for the first three approximations $(j = 0,1,2)$.

When $j = 0$:

$$U^{1,0}_{\xi\xi} + U^{1,0}_{\eta\eta} = 0;\ qV^{1,0}_{\eta\eta} + mU^{1,0}_{\xi\eta} = 0;$$
$$-a_{11}U^{1,0}_{\xi\eta} + \varepsilon b_{11}\phi^{1,0}_{\eta\eta} = 0;$$

When $j = 1$:

$$U^{1,1}_{\xi\xi} + U^{1,1}_{\eta\eta} = 0;\ qV^{1,1}_{\eta\eta} + mU^{1,1}_{\xi\eta} = 0;$$
$$-a_{11}U^{1,1}_{\xi\eta} + \varepsilon b_{11}\phi^{1,1}_{\eta\eta} = 0; \tag{9}$$

When $j = 2$:

$$U^{1,2}_{\xi\xi} + U^{1,2}_{\eta\eta} - a_{11}\phi^{1,0}_{\xi\eta} = 0;\ qV^{1,2}_{\eta\eta} + mU^{1,2}_{\xi\eta} = 0;$$
$$-a_{11}U^{1,2}_{\xi\eta} + \varepsilon b_{11}\phi^{1,2}_{\eta\eta} = \quad .$$

Here and further it is assumed that differentiations (indices ξ, η) are performed according to those coordinates ξ_n, η_n $(n = 1,2)$, whose indices coincide with the first superscripts of the functions.

After substituting transformations (8) into system (5) using the appropriate expansions and splitting by the $\varepsilon^{1/2}$ parameter we obtain an infinite system of equations concerning the functions $V^{2,j}$, $V^{2,j}$, $\phi^{2,j}$ $(j = 0,1,...)$, which determine the solutions of the second type.

It follows from system (9) that in the first two approximations $(j = 0,1)$ the main functions $U^{1,j}$ ($V^{2,j}$ for the second stress state) are determined from Laplace's equations (with $q = 1$ or obvious replacement of one of the variables), and the auxiliary functions are expressed by simple integration over the main ones.

In the third approximation $(j = 2)$ and further, for the stressed state of the first type, the functions $U^{1,j}$ are found from the Poisson equation with the known right-hand side, which contains only the ϕ function found in the previous approximations. A similar situation occurs in the tense state of the second type for the $V^{2,j}$ function but starting from the fourth approximation $U^{2,j}$ and beyond.

It can be seen from relations (9), that the stress-strain states of the first and second types are connected only through boundary conditions. Since the main functions $U^{2,j}$, $V^{2,j}$ are determined from the Laplace (Poisson) equations, the effectiveness of the method depends on whether it is possible to formulate appropriate boundary value problems for finding these functions.

In cases where only electrical interaction is at play, the displacement vector components will be notably lesser compared to those in mechanical loading scenarios.

However, in instances of concurrent mechanical and electrical (or magnetic) loading, linearity permits the separate consideration of these two problems (comprising three distinct stress states). The comprehensive solution can then be expressed as an aggregate of solutions to individual problems. The generalization of the asymptotic method to two-dimensional problems of electro elasticity is verified for *individual model problems.*

1. Let $x \geq 0$ a normal concentrated force act on the boundary of the half-plane $|y| < \infty$ at the origin of the coordinates, $(0;0)$ there are no tangential stresses $x = 0$, and at infinity, the derivatives of the components of the displacement vector turn to zero, i.e. the boundary conditions have the form:

$$\sigma_1 = -P_0\delta(y),\ (x = 0), \qquad (10)$$

at infinity, all terms are zero. Here $\delta(y)$ - is the Dirac delta function.

In the first approximation, we come to the integration of equations (5) with the following boundary conditions:

$$U_{0x} = -(P_0 / B_1)\delta(y) \text{ at } x = 0,$$

the derivatives converge to zero at infinity, and the equations for the components of the induction $D_{1,0}$ vector, $D_{2,0}$ and the electric field intensity $E_{1,0}$ vector, $E_{2,0}$ with the boundary conditions:

$$V_{0x} = -U_{0y} \text{ at } x = 0,$$

the derivatives at infinity converge to zero.

By sequentially solving the indicated boundary value problems, we obtain

$$\frac{\partial U_0}{\partial x} = -\frac{P_0}{B_1}\frac{\omega_1^{-1}x}{\pi\left(\omega_1^{-2}x^2 + y^2\right)}, \frac{\partial U_0}{\partial x} = -\frac{P_0}{B_1}\frac{\omega_1}{\pi}\frac{y}{\left(\omega_1^{-2}x^2 + y^2\right)},$$

$$\frac{\partial V_0}{\partial x} = \frac{P_0}{B_1}\frac{\omega_1}{\pi}\frac{y}{\left(\omega_2^2x^2 + y^2\right)}, \frac{\partial V_0}{\partial y} = -\frac{P_0}{B_1}\frac{\omega_1}{\pi}\frac{x}{\left(\omega_2^2x^2 + y^2\right)},$$

where $\omega_1^2 = B_1/G$, $\omega_2^2 = B_2/G$.

Since $\tau = G\left(U_{0y} + V_{0x}\right)$, and $\lim\limits_{\omega_1^{-1}x \to 0+0} \frac{\omega_1^{-1}x}{\pi\left(\omega_1^{-2}x^2 + y^2\right)} = \delta(y)$, the boundary conditions (10) are fully satisfied.

In subsequent approximations when resolving analogous problems, errors stemming from auxiliary functions exhibit a higher degree of insignificance and are consequently rectified.

If on the boundary of the half-plane $x \geq 0$, $|y| < \infty$ the induction vector of the electric field at the origin of the coordinates is given in the following form

$$D_1 = \varphi_y^E = d_1^0\delta(y),\quad D_2 = -\varphi_x^E = d_2^0\delta(y),\quad (x = 0),$$

then the solution of equation (2) under the specified boundary conditions has the form

$$D_1 = \frac{1}{\pi k}\frac{d_1^0k^2x - d_2^0y}{k^2x + y^2},\quad D_2 = \frac{k}{\pi}\frac{d_1^0y + d_2^0x}{k^2x + y^2},\quad k^2 = \frac{B_1b_{11}}{Gb_{22}}.$$

In the case of the function φ^E on the boundary of the half-plane as $\varphi^E = \varphi_0\delta(y)$, $(x = 0)$ one gets

$$\varphi^E(x, y) = \frac{\varphi_0 kx}{\pi\left(k^2x^2 + y^2\right)},$$

$$D_1 = -\frac{\varphi_0 2k}{\pi}\frac{xy}{\left(k^2x^2 + y^2\right)^2},\quad D_2 = \frac{\varphi_0 k}{\pi}\frac{k^2x^2 - y^2}{\left(k^2x^2 + y^2\right)^2}.$$

Other unknowns are searched for according to the appropriate formulas.

Numerical analysis. For the numerical analysis the materials with the following characteristics were chosen

$$c_{44}^{(1)} = 43.7\cdot 10^9\,Pa,\ e_{15}^{(1)} = 17\frac{C}{m^2},\ \alpha_{11}^{(1)} = 15.1\cdot 10^{-9}\frac{C}{V\cdot m},$$

$$d_{11}^{(1)} = 0,\ h_{15}^{(1)} = 165\frac{N}{a\,m},\ \gamma_{11}^{(1)} = 180.5\cdot 10^{-6}\frac{N\cdot s^2}{C^2},$$

$$c_{44}^{(2)} = 42.47\cdot 10^9\,Pa$$

$$e_{15}^{(2)} = -0.48\frac{C}{m^2},\ \alpha_{11}^{(2)} = 0.0757\cdot 10^{-9}\frac{C}{V\cdot m},\ d_{11}^{(2)} = 0,$$

$$h_{15}^{(2)} = 385\frac{N}{a\,m}\quad \gamma_{11}^{(2)} = 414.5\cdot 10^{-6}\frac{N\cdot s^2}{C^2}$$

The crack sliding for the electric field at the infinity $E^\infty = 9\cdot 10^3 V/m$, the magnetic field at the infinity $H^\infty = 0$ and $E^\infty = 0$, $H^\infty = 1.7\cdot 10^4\ A/m$ and different values of the mechanical loading at the infinity σ^∞ [Pa] are presented in Figures 5.2a and 5.2b, respectively. It can be seen from these Figures that even for zero mechanical loading σ^∞ the crack faces slide with respect to each other due to nonzero electric(Fig. 5.2a) or magnetic (Fig. 5.2b) fields.

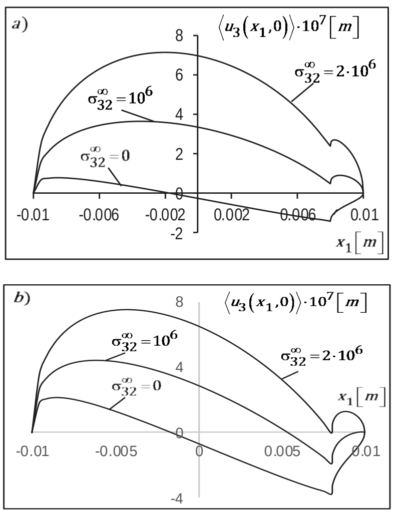

Fig. 5.2. The displacement jump at the segment $[c,b]$ for $E_1^\infty = 9\cdot 10^3[V/m]$, $H_1^\infty = 0$ (a) and $E_1^\infty = 0$, $H_1^\infty = 1.7\cdot 10^4[A/m]$ (b)

Variations of the normalized stress intensity factor (SIF) K_3 is shown in Tables 5.1 for $\sigma_{23}^{(1)} = 10^6\ Pa$, $E_1^\infty = 0$ and different values of the relative length of the contact zone λ and magnetic fiedl H_1^∞. It can be seen from this Table that for each λ the decreasing of magnetic field H_1^∞ (growing it on modules) leads to decreasing of the SIF K_3 and even to turning it into zero for $\lambda = 0.1$ and $H_1^\infty = -18742\,A/m$. It means

Table 5.1

Variations of the normalized stress intensity factor (SIF) K_3 with respect to λ and H_1^{∞}

H^{∞} / λ	**0**	**-5000 A/m**	**-10000 A/m**	**-15000 A/m**	**-18742.6 A/m**
0.1	177676.0	130277.0	82877.8	35478.5	0.0
0.2	181031.0	150716.0	120401.0	90086.0	67394.7
0.3	181731.0	160997.0	140263.0	119529.0	104009.0

that electric and magnetic fields can be used for governing of the SIF and decreasing the probability of fracture.

Conclusions and prospects for further development in this direction. The presented method, employing a generalized perturbation approach for solving electroelasticity problems, demonstrates the feasibility of formulating boundary value problems for key functions. This allows the original electroelasticity problem to be expressed as a superposition of simpler boundary value problems, where mechanical and electrical components can be separated, yet they interact through boundary conditions. This method broadens the scope for investigating new and highly relevant electroelasticity problems, extending the small parameter method to two-dimensional electroelasticity problems. Consequently, the approach enables preliminary assessments of stress-strain states in structures or components under different contact conditions, offering analytical solutions to various practical problems. Additionally, by employing segmented analytical functions to represent field components along piezoelectric material interfaces, the problem is simplified into a boundary value problem, enabling accurate analytical solutions. These solutions form the basis for deriving analytical expressions for stress tensor components, electric and magnetic field induction vectors, displacement discontinuities, and electric and magnetic field potentials at specific segments of the material interface. The outcomes are compared across different crack models, identifying critical parameters influencing failure under diverse loading scenarios. Furthermore, the study explores shielding phenomena at the interfacial crack tip within piezoelectric materials, highlighting how the influence of electric and magnetic fields on key field characteristics surrounding the crack apex varies with the magnitude of the external load. Numerical implementation for the antiplane scenario is presented in current paper for the very first time.

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